

VLSI CMOS

Abstraction device $\rightarrow$ circuit $\rightarrow$ gate $\rightarrow$ functional $\rightarrow$ behavioral model

Problem: power distribution integrity (V_{DD}); reliability; noise; interference

A vs D NM

A more energy efficient, less reliable

Material Si vs 3S vs wide-bandgap (SiC, GaN)

MOS vs BJT power

Semi $T \uparrow$, e^- jump to $E_c \rightarrow$ EHP

$$np = n_i^2$$

Fermi Dirac $f(E) = \frac{1}{1 + e^{\frac{E - E_f}{kT}}}$

$$n = n_i e^{\frac{E_f - E_i}{kT}} \quad p = n_i e^{\frac{E_i - E_f}{kT}}$$

pnj band bend $\leftarrow E \rightarrow \Delta V$

diffuse $\rightarrow q^{+/-}$

$$\frac{dE}{dx} = \frac{p}{K_s \epsilon_0}$$

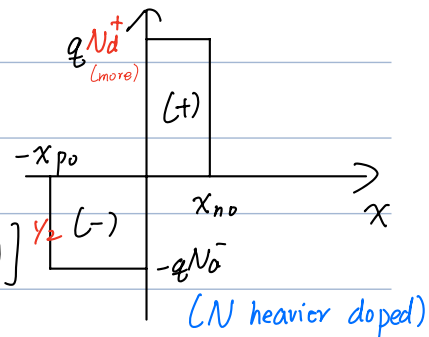
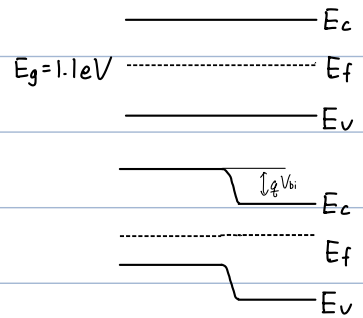
$$K_s = 11.7, \epsilon_0 = 8.85 \times 10^{-14}$$

$$\int_0^{E_{max}} dE = \int_{x_p}^0 \frac{p}{K_s \epsilon_0} dx$$

lin $E_{max} = \frac{q N_A}{K \epsilon_0} x_p = \frac{q N_D}{K \epsilon_0} x_n$

$$V_{bi} = \frac{1}{q} (kT \ln \frac{N_A}{n_i} + kT \ln \frac{N_D}{n_i}) = \frac{kT}{q} \ln \left(\frac{N_A N_D}{n_i^2} \right) = \frac{1}{2} E_{max} W$$

$$W = \left[\frac{2 K_s \epsilon_0 V_{bi}}{q} \left(\frac{1}{N_A} + \frac{1}{N_D} \right) \right]$$



bias V_A : non-eq

$$I \sim e^{\frac{q V_A}{kT}}$$

C. $C_o = \frac{K_s \epsilon_0}{W}$ (per area, ss, $\downarrow$ as RB $\uparrow$, parasitic)

MOS/FET SD sym., depends on how biased in circuit

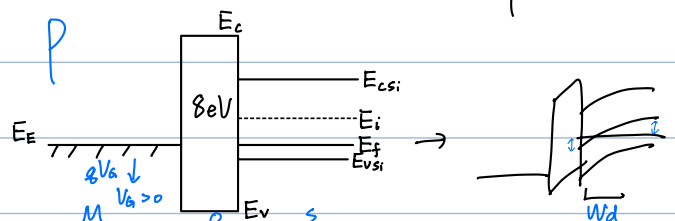
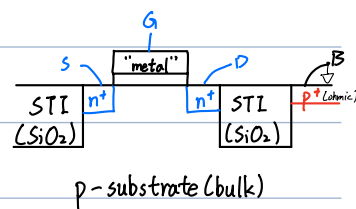
$V_A = 0$, flat band V_{FB} to ensure $E = 0$

$V_A > 0$, band bend $\rightarrow$ depletion in S

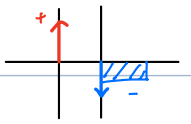
$$E_i - E_f = kT \ln \left(\frac{N_A}{n_i} \right) \equiv \phi_f$$

@ interface @ bulk

Once $E_f - E_i = E_i - E_f \rightarrow$ inversion



past V_T ↓
dep. region no longer grows. e^- grows in inv. to balance



$$V_{ov} = V_G - V_T \text{ decides current}$$

$$W_d = \sqrt{\frac{2K_s \epsilon_0 \phi_s}{q N_A}} \quad \phi_s: \text{surface vs bulk};$$

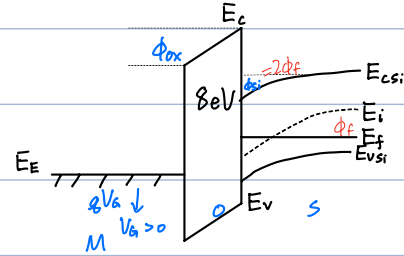
$$W_d^{\max} = \sqrt{\frac{2K_s \epsilon_0 2|\phi_f|}{q N_A}} \quad Q_{\text{blk}} = q N_A W_d^{\max}$$

$$V_T = \phi_{ox} + \phi_s$$

$$= \frac{Q_{\text{bulk}}}{C_{ox}} + 2|\phi_f| + V_{FB}$$

compensate for non-flat-band

$$C_{ox} = \frac{3.9 K_{ox} \epsilon_0}{t_{ox}}$$



Use nonuniform doping profile to control W_d and V_T

As size ↓, # dopants countable (random dopant fluctuation) → :C

sln. deplete Si → no dopant → engineer ϕ_m for a good V_{FB}

Mobility $v_{\text{drift}} = \mu E \rightarrow v_{\text{sat}} \approx 10^7 \text{ cm/s}$

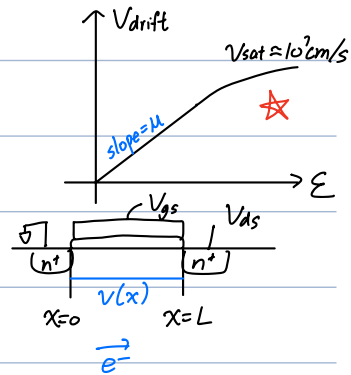
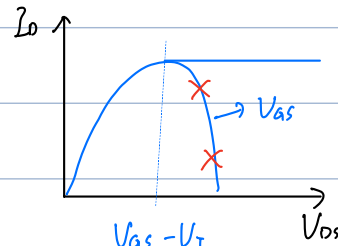
long-channel Schokley (assume ↓)

$$I_D = W Q_i v_{\text{drift}}$$

$$= W C_{ox} (V_{gs} - V(x) - V_T) (\mu \frac{dv}{dx})$$

$$\int_0^L I_D dx = \int_0^{V_{ds}} W C_{ox} (V_{gs} - V_T - V(x)) \mu dv$$

$$I_D = \frac{W}{L} \mu C_{ox} \left[(V_{gs} - V_T) V_{ds} - \frac{V_{ds}^2}{2} \right] \text{ (linear)}$$



$V_{ds} > V_{ov}$ pinchoff (no free carriers, e^- swept to drain) → $I_{\text{sat}} = \frac{W}{2L} \mu C_{ox} (V_{gs} - V_T)^2$

$$I_{\text{sat}} \sim V_{ov}^2, \text{ since } Q_i \propto V_{\text{drift}}$$

short channel $Q_i \propto V_{\text{sat}} \rightarrow \sim V_{ov}$

$$v_{\text{drift}} = \frac{\mu E}{1 + E/E_{\text{crit}}} \sim 10^4 \text{ V/cm} \quad \text{short}$$

$$I_D = W C_{ox} (V_{gs} - V(x) - V_T) \frac{\mu E}{1 + E/E_{\text{crit}}}$$

$$I_D = \frac{W}{L} \frac{\mu C_{ox}}{1 + \frac{V_{ds}}{E_{\text{crit}} L}} \left[(V_{gs} - V_T) V_{ds} - \frac{V_{ds}^2}{2} \right]$$

$$\frac{\partial I_D}{\partial V_{ds}} = 0 \rightarrow V_{ds} = V_{\text{dsat}} = \frac{2(V_{gs} - V_T)}{1 + \sqrt{1 + \frac{2\mu(V_{gs} - V_T)}{E_{\text{crit}} L}}}$$

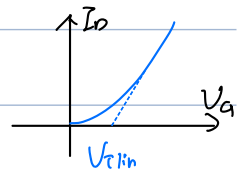
$$I_{\text{dsat}} = C_{ox} W V_{\text{sat}} (V_{gs} - V_T) \sqrt{\frac{V_{gs} - V_T}{L}} \quad \text{if } \frac{V_{gs} - V_T}{L} \gg \frac{E_{\text{crit}}}{2}, I_{\text{dsat}} \rightarrow C_{ox} W V_{\text{sat}} (V_{gs} - V_T) \quad \text{lin}$$

Enhancement : off when $V_G = 0$ ($V_T < 0$ for p, > 0 for n) TAOH?

Depletion on (rare)

V_T 1. device physical: onset of strong inv.

def 2. triode $I_D = \sim (V_{GS} - V_T) V_{DS}$ (extrapolate lin. part $\rightarrow V_{T,lin}$)



3. sat. find V_{GS} s.t. I_D reaches a specific density when $V_{DS} = V_{DD}$ (E. $\frac{1 \mu A}{\mu m}$)

Scaling (Dennard) $w \rightarrow \frac{w}{K}$; $L \rightarrow \frac{L}{K}$; $t_{ox} \rightarrow \frac{t_{ox}}{K}$; $V \rightarrow \frac{V}{K}$ (const. E scaling)

1. Depletion region $W_{dep} \sim \sqrt{\frac{V}{N}} \sim \frac{1}{K} \rightarrow$ want $N \rightarrow KN$

but $V = V_{bias} + V_{built-in} \sim$ bandgap $\rightarrow$ can't scale $\rightarrow$ short channel

2. C $C_{gate} = \frac{\epsilon w L}{t_{ox}} \sim \frac{1}{K}$

inv. layer charge density

3. $Q_i \sim \frac{\epsilon V}{t_{ox}} \sim 1$, but $V \sim \frac{1}{K}$

4. $I_D \sim Q_i W_{eff} \sim \frac{1}{K}$

5. delay $\approx \frac{CV}{I} \sim \frac{1}{K} \rightarrow f \sim K$

6. power $= IV \sim \frac{1}{K^2}$ (density ~ 1)

V_T inability $I_D \sim e^{\frac{q(V_{GS} - V_T)}{m k T}}$

$\log_{10}$ slope $= \frac{m k T}{q} \frac{1}{\log_{10} e} \approx 60 \text{ mV/dec}$ of I_D

if $V_T \downarrow \rightarrow I_{leak} \uparrow$

if size $\downarrow$, but $W_{dep} \nrightarrow \rightarrow D$ starts to control Q_{dep}

$V_T = V_{FB} + 2|\Phi_F| + \frac{Q_{bulk}}{C_{ox}}$ not much control

If $V_{DS} \uparrow$, $V_{T,eff} \downarrow$, so $I_D \uparrow$ slightly $\rightarrow$ short channel effect

(DIBL: drain Induced Barrier Lowering $\sim \frac{mV}{V}$)

V_T rolloff if L too short, $V_T \downarrow$, same reason

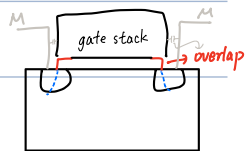
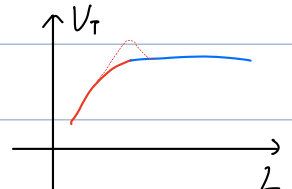
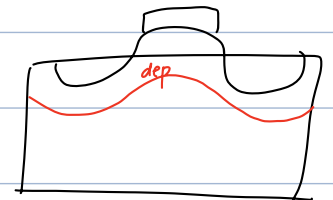
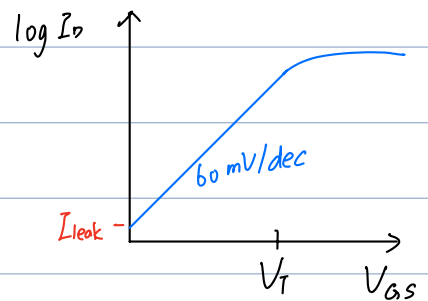
want minimal variation

Cap from device/wires (0.1 fF/ μm)

intrinsic: $C_{gate} \sim I_D$

extrinsic (parasitic) — overlap from V_{GS} , V_{GD} ; also between M contact & gate stack

diffusion (junction): jct C across W_{dep} , controlled by layout

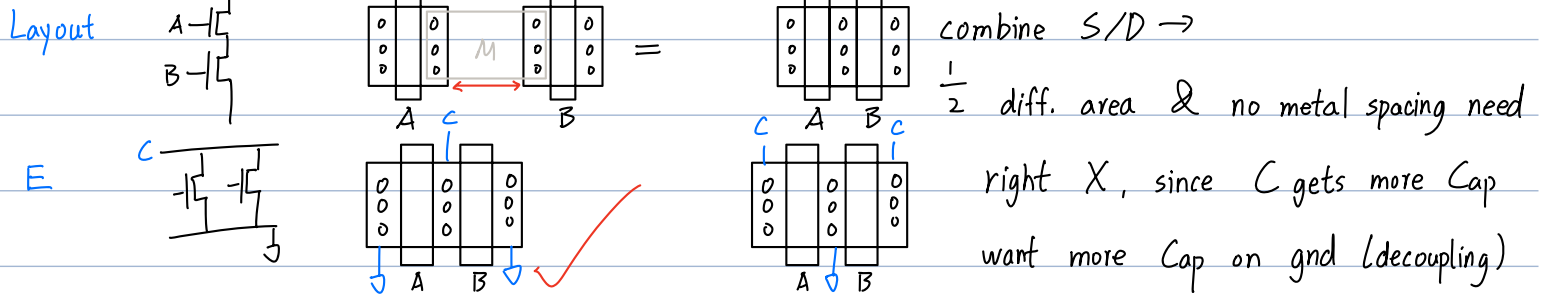


Intrinsic distribution C_{gs} C_{gd} C_{gb}

cutoff no inv. $\rightarrow C_{gs} = C_{gd} = 0$, $C_{gb} = C_{ox} w L \sim C_{bulk}$ series w/

triode uniform inv. $\rightarrow C_{gs} = C_{gd} = \frac{1}{2} C_{ox} w L$; $C_{gb} = 0$

sat. most C not at drain $\rightarrow C_{gs} = \frac{2}{3} C_{ox} w L$; $C_{gd} = C_{gb} = 0$



Body effect if $V_{SB} \neq 0$

$$V_T = V_{FB} + 2|\phi_F| + \frac{Q_B}{C_{ox}} ; Q_B = \sqrt{2q\epsilon_{si}N_A 2|\phi_F|}$$

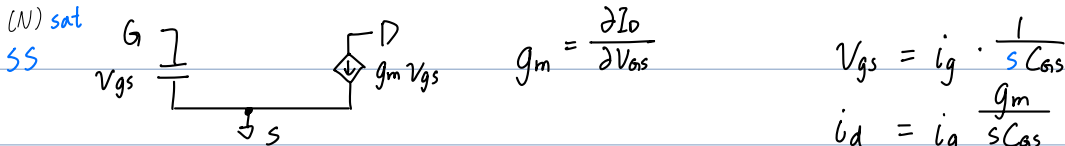
$2\phi_F + V_{SB} \rightarrow V_T \uparrow$

must apply larger V_G to create inv. May also use V_{SB} to control V_T

Transfer curve

f_T : unity current gain cutoff f

F_{04} } delay of D0 driving 4 D0s of same size
 F_{01} }



$$V_{GS} = i_g \cdot \frac{1}{sC_{GS}}$$

$$i_d = i_g \cdot \frac{g_m}{sC_{GS}}$$

$$|A_i(j\omega)| = \frac{g_m}{\omega C_{GS}} = 1$$

$$f_T = \frac{1}{2\pi} \frac{g_m}{C_{GS}} = \frac{1}{2\pi} \frac{W C_{ox} V_{sat}}{W L C_{ox}}$$

(transit time) $= \frac{1}{2\pi} \frac{V_{sat}}{L} \equiv \frac{1}{2\pi} \cdot \frac{1}{\tau_{tr}}$

F_{01} $V_{sat,p} \approx \frac{1}{2} V_{sat,n}$, so $W_p = 2W_n$ (but ~ same for modern)

N $t \approx \frac{V_{DD}}{I_{sat}} (C_{gn} + C_{dn})$ $\eta \equiv \frac{C_g}{C_d} \sim 1.5 \sim 2$

$$= \frac{V_{DD}}{I_{sat}} C_{gn} (1 + \frac{1}{\eta})$$

$$= \frac{V_{DD} - W L C_{ox} (1 + \frac{1}{\eta})}{W C_{ox} (V_{as} - V_T) V_{sat}}$$

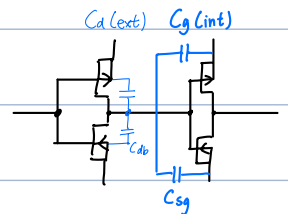
$$= \frac{L}{V_{sat}} (1 + \frac{1}{\eta})$$

$$= \tau_{tr} (1 + \frac{1}{\eta})$$

$P+N$: $t(2w) \equiv 3 \tau_{tr} (1 + \frac{1}{\eta}) \equiv \tau_{inv}$ $\eta = \frac{C_{gn} \times (2+1)}{C_{dn} \times 2+1}$ (p+n)

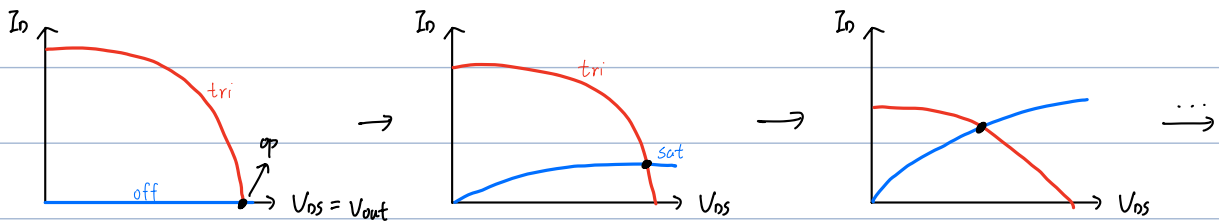
$F_{04} = \frac{4C_g + C_d}{C_g + C_d} \cdot F_{01}$ (= x4 if $\eta = \infty$)

$$\equiv (1 + 4\eta) \cdot \frac{3}{\eta} \tau_{tr} = 3 \tau_{tr} (4 + \frac{1}{\eta})$$



(s are not switching, irrelevant)

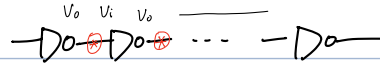
Noise (DC)



rapid change in middle $\rightarrow$ Do trans. curve

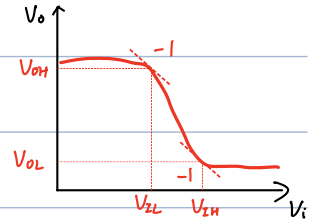
Noise at end will be **restored**

NM (DC, acting on every **node**)



+ worst-case DC noise (*)

if AC (NM_{AC}), will be filtered by C_g (better)



$$NM_L = (V_{ZL} - V_{OL}) ; NM_H = V_{OH} - V_{ZH}$$

Switching energy output charges C_L

$$\begin{aligned} 0 \rightarrow 1 \quad E &= \int dt i_{DD}(t) V_{DD} \\ &= V_{DD} \int_0^\infty dt C_L \frac{dV_{out}}{dt} \\ &= C_L V_{DD}^2 \end{aligned}$$

but $E_c = \frac{1}{2} C_L V_{DD}^2 \rightarrow$ rest heat

$1 \rightarrow 0$ stored $\frac{1}{2}$ diss.

$$P = C_L V_{DD}^2 f$$

I_{crossover}: while switching, a transient time where **both Q on** ($\sim 10-20\%$ p)

especially when V_{in} switch **slow** (low slow rate)

spice

E. channel length

E. in a water

Compact model: many params. Have a distribution (may be correlated)

corner model: set 3σ from $\mu \rightarrow$ ensure still work

TT, SS, FF, SF, FS $\rightarrow$ corners

PVT: process, voltage, temperature

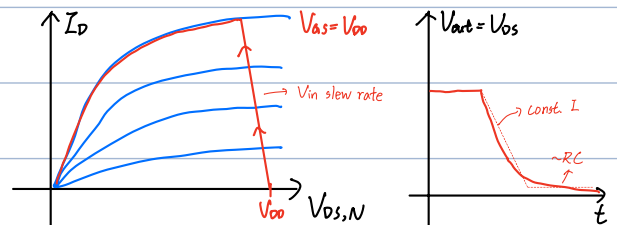
Delay $V_{in}: 0 \rightarrow 1$

$\sim$ saturated ramp. delay T_{PHL} , $T_{PLH} = \Delta t$ for 50% V_{DD}

slow $\sim$ ramp slope of $10\% - 90\%$

or extrapolate to $0 \sim 100$ ($\div 0.8$), up to ur def

$$C_g \left[\frac{fE}{\lambda_{um}} \right] @ \text{min length}$$



$$R_{nFET} = W_n R_{p\text{pulldown}} \quad [k\Omega \cdot \mu m]$$

$$R_{pFET} = W_p R_{p\text{pullup}}$$

$$C_g = C_{gn} = C_{gp} = \frac{C_{gate}}{W} \approx \frac{K \epsilon_0 L}{t_{ox}}$$

$$\left[\frac{fF}{\mu m} \right]$$

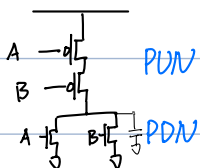
@ min width

For p: 2W; n: W inverter: **logical effort** $g_{inv} \equiv 1$

$$\text{electrical effort } h_{inv} \equiv \frac{C_{out}}{C_{in}} = 1$$

$$\text{normalized delay } F \equiv \frac{\tau_{logic\ gate}}{\tau_{inv}} = g_{gate} \cdot h_{gate}$$

E. nor

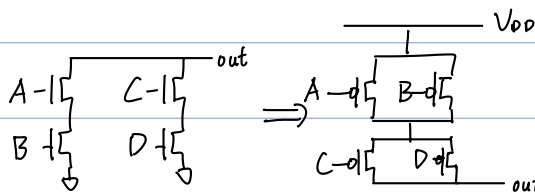


exactly 1 on at a time

want input w/ more delay to be closer to output

Ser. in PDN $\leftrightarrow$ || in PUN

want ≤ 2 Q high per network



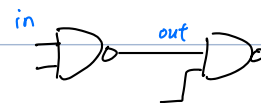
$$\tau_{inv} = FOI_{inv} = \frac{R_{PFET}}{W} \cdot 3WC_g = 3C_g R_{nFET}$$

E. NAND size wrt to inv. $\rightarrow$ p: 2W; n: 2W

$$\tau_{nand} = FOI_{nand} = (4WC_g) \times \left(\frac{R_{nfet}}{2W} \right) \cdot 2 = 4C_g R_{nfet}$$

$$f \equiv \frac{\tau_{nand}}{\tau_{inv}} = \frac{4}{3}$$

$$h = \frac{4WC_g}{4WC_g} = 1 \rightarrow g_{nand} = \frac{f}{h} = \frac{4}{3}$$



E2. NOR $\rightarrow$ p: 4W, n: 1W

$$\tau_{nor} = FOI_{nor} = (5WC_g) \times \left(\frac{R_{nfet}}{W} \right) = 5C_g R_{nfet}$$

$$d \equiv \frac{5}{3}, g_{nor} = \frac{5}{3}$$

E. if NOR driving 2 NORs w/ same 4W - W

$$FOI_{nor} = (10WC_g) \times \left(\frac{R_{nfet}}{W} \right) = 10C_g R_{nfet}$$

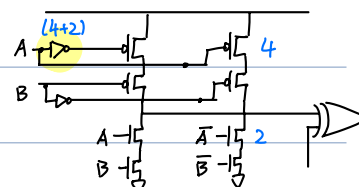
$$d = \frac{10}{3}, h = 2, g = \frac{5}{3} \quad (\text{some logical, more electrical})$$

E3. XOR Need to size $\rightarrow$

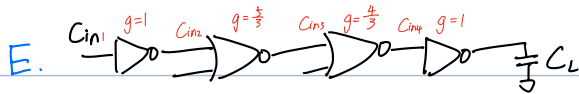
$$FOI_{xor} = FOI_{main} \times FOI_{inv} \quad \text{multiply stage effort}$$

$$= (12WC_g) \left(\frac{R_{nfet}}{2W} \cdot 2 \right) \times (6WC_g) \left(\frac{R_{nfet}}{2W} \right)$$

$$= 12C_g R_{nfet} \times 3C_g R_{nfet} = 4\tau_{inv} \cdot 1\tau_{inv} \rightarrow F = 4$$



$$\text{E. if inv is } 2W/W, FOI_{xor} = (9WC_g) \dots \times (6WC_g) \left(\frac{R_{nfet}}{W} \right) = 3\tau_{inv} \cdot 2\tau_{inv} \rightarrow F = 6 > 4 \therefore$$



Assume C_{in}, C_w fixed (if not, make $C_{in} \rightarrow \infty$)

$$F = \prod_{i=1}^n F_i = \prod_{i=1}^n (g_i h_i) = \prod_{i=1}^n g_i \cdot \prod_{i=1}^n h_i = \left(\frac{20}{9}\right) \left(\frac{C_{in2}}{C_{in1}} \times \dots \times \frac{C_L}{C_{in4}}\right) = \frac{20}{9} \frac{C_L}{C_{in1}} \rightarrow \text{fixed}$$

$$\tau_{path} = \sum_{i=1}^n (g_i h_i) \tau_{inv} \rightarrow \text{minimizable}$$

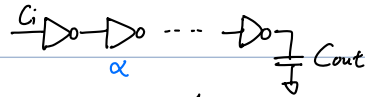
E. just one NOR, one NAND

Let norm. delay $y \equiv f_{nor} + f_{nand} = \frac{F}{f_{nand}} + f_{nand} = f_{nand} \left(1 + \frac{F}{f_{nand}^2}\right)$

$$\frac{\partial y}{\partial f_{nand}} = 1 - \frac{F}{f_{nand}^2} = 0 \rightarrow f_{nand} = \sqrt[2]{F}$$

General for n stages: $f_i = \sqrt[n]{F}$ for min delay (if no branching (const. H))

+ stages N -stage inv. chain, $\eta = \infty$, $f_i = \alpha$



$$F = \prod_{i=1}^N f_i = \alpha^N = \frac{C_{out}}{C_{in}}$$

$$\tau_d = \tau_{inv} \sum_{i=1}^N F = \tau_{inv} \alpha N$$

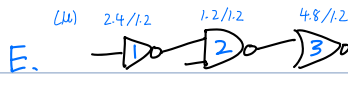
$$N = \frac{\ln(C_{out}/C_{in})}{\ln \alpha}$$

$$= \tau_{inv} \alpha \frac{\ln(C_{out}/C_{in})}{\ln \alpha}$$

$$0 = \frac{\partial \tau_d}{\partial \alpha} =$$

$$\alpha = e \quad (\text{closest even})$$

if include C_d , $\alpha = 4$

E.  $C_g = \frac{2fF}{\mu m}$, $R_{pf} = 16 \text{ k}\Omega \cdot \mu m$, $R_{nf} = 8 \text{ k}\Omega \cdot \mu m$, $\tau_{inv} = 3R_{nf}C_g = 48 \text{ ps}$

① find eff C_{out} , $f_3 = \frac{500 \mu m}{3 \cdot 1.2 \mu m} = \frac{5}{3} \cdot \frac{1pF}{2fF \cdot 6 \mu m}$

$$f_2 = \frac{6 \mu m}{3 \cdot 0.6 \mu m}$$

$$f_1 = \frac{2.4 \mu m}{3 \cdot 1.2 \mu m}$$

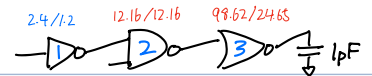
② $\tau_{delay} = \tau_{inv} (f_3 + f_2 + f_1) = 6858 \text{ ps}$, too long

③ $F = f_3 f_2 f_1 = 308.6 \rightarrow f_{op} = \sqrt[3]{F} \approx 6.76 \rightarrow \tau_{delay}' = 973 \text{ ps}$

④ $W_{inv, eff, 3} = \frac{500 \mu m}{3 \cdot 6.76} \approx 24.65 \mu m \rightarrow 4W/W = 98.62/24.65$

$W_{inv, eff, 2} = \frac{5 \cdot 24.65 \mu m}{3 \cdot 6.76} = 6.08 \mu m \rightarrow 2W/2W = 12.16/12.16$

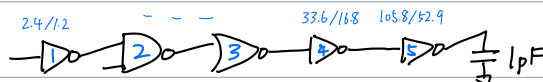
$W_{inv, eff, 1} = \frac{4 \cdot 6.08 \mu m}{3 \cdot 6.76} = 1.2 \mu m \rightarrow$



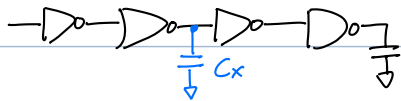
E': add 2 $\rightarrow f_{op} = \sqrt[5]{F} \approx 3.15$

(if other gate added, $g \uparrow \rightarrow f \uparrow$)

$$\tau_{delay}' = 756 \text{ ps}$$

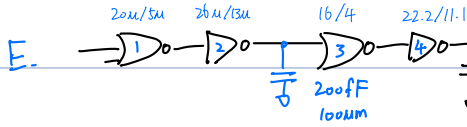


Branching (fanout)



Add $C_x \rightarrow h$ no longer const. $H = \frac{C_L}{C_{in4}} \cdot \frac{C_{in4}}{C_{in3}} \cdot \frac{C_{in3} + C_x}{C_{in2}} \cdot \frac{C_{in2}}{C_{in1}}$ logical G unaffected

$$= \frac{C_L}{C_{in1}} \cdot \frac{C_{in3} + C_x}{C_{in3}} \quad (\text{if } C_{in3} \gg C_x, \text{ ideal})$$



$$C_g = 2 \frac{fF}{\mu m}$$

size up the following gate

$$f_1 \equiv 2.6, f_2 \equiv 3, f_3 \equiv 3, f_4 \equiv 3 \quad \text{w/o } C_x$$

$$11.7 \tau_{inv}$$

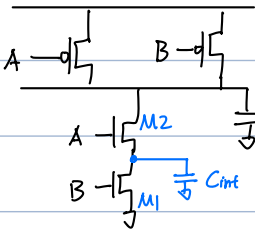
if $1.5 \times f_3 \rightarrow f'_2 = 3.25, f'_3 = 1.83$, better! (want $C_{in3} \uparrow$)

$$10.78 \tau_{inv}$$

$\hookrightarrow 4: 30/15 \rightarrow f'_{2:4} = 3.25, 2.5, 2.27, \checkmark$

$$10.63 \tau_{inv}$$

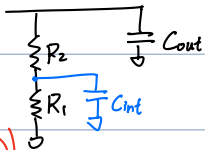
Stacks



if $A=1, B=0 \rightarrow 1$

C_{int} needs to be discharged first before M2 can be on

B slower, delay $\propto (\text{stack level})^2$



C_{int} only discharges via R_1 .

$$T_{elmore} = R_1 C_{int} + (R_1 + R_2) C_{out}$$

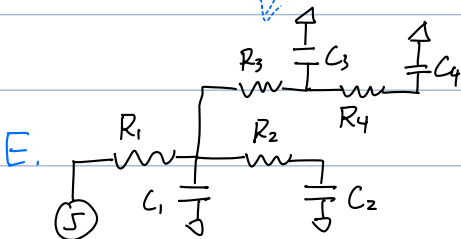
(add $R_i C_i$ for each dev. in stack)

$$T_D = \sum_{\text{node } i} R_{i \rightarrow \text{src}} C_i$$

only valid for 1. single driving node &

2. all caps are gnded (path to R gnd)

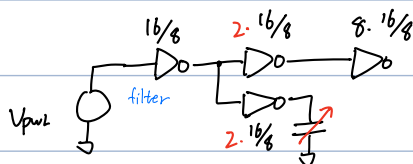
3. no resistive loops



$$T_D = C_1 R_1 + C_2 (R_1 + R_2) + C_3 (R_1 + R_3) + C_4 (R_4 + R_3 + R_1)$$

PS3 C

want F_{04}

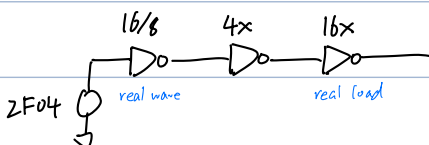


$$\text{slew time} = 2 F_{04}$$

vary C_L to get some delay

$$C_g = \frac{C_L}{8(16+8)\mu m}$$

R



$$R_{nfet} = \frac{t_{pHL}}{C_L W_L} \times W_{nfet}$$

$$R_{pfet} = \frac{t_{pUH}}{C_L W_L} \times W_{pfet}$$