Freq LT → H(s)

p/z: if some p has ≥0 real part, unstable

z: no stability eff, changes I/O effect

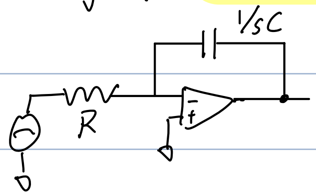
H(s) → impulse/step resp., natural/forced, zi/zs

→ $H(j\omega)$ → bode plt

$x(t) = A \sin(\omega t)$

$y(t) = |H(j\omega)| \sin(\omega t + \angle H(j\omega))$

E. ∫

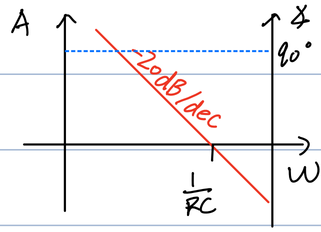


$V_{out}(s) = 0 - \frac{1}{sC} \cdot V_{in} \cdot \frac{1}{R}$

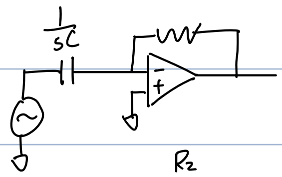
$H(s) = -\frac{1}{sRC}$

$H(j\omega) = \frac{j}{\omega RC}$

p: $s=0$



E. $\frac{d}{dx}$

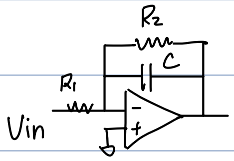


$H(s) \overset{\dots}{=} -sRC$

+ 20 dB/dec, -90°

z: $s=0$

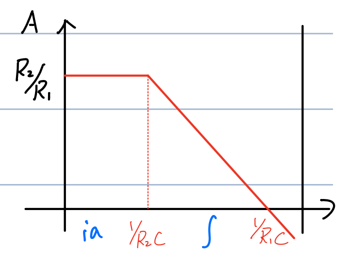
E. LPF



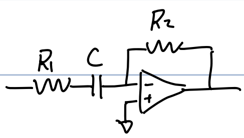
$H(s) = -\frac{R_2}{R_1} \frac{1}{1+sR_2C}$ p: $s = -\frac{1}{R_2C}$

ia → ideal ∫

-180° → -270° (+90°)



E. HPF



$H(s) = -\frac{R_2}{R_1} \frac{sR_1C}{1+sR_1C}$

ideal $\frac{d}{dx}$ → ia

-90° → -180°

1p|z

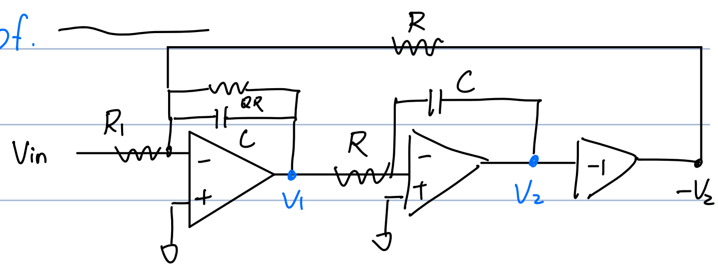
E. BPF (U)

$H(s) = -\frac{R_2}{R_1} \frac{sR_1C_1}{1+sR_1C_1} \frac{1}{1+sR_2C_2}$

2p|z

-90° → -180° → -270°

E. HW2 All comp. give same pole, but diff. zero pf. ______



$\frac{V_{in}}{R} + \frac{V_1}{QR} + sCV_1 - \frac{V_2}{R} = 0$

$sCV_2 + \frac{V_1}{R} = 0$

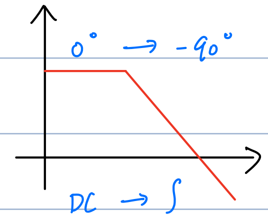
$$\begin{bmatrix} \frac{1}{QR}+sC & -\frac{1}{R} \\ \frac{1}{R} & sC \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} -\frac{1}{R_1} \\ 0 \end{bmatrix} V_{in}(s)$$

$$V_1(s) = \frac{\begin{vmatrix} -\frac{1}{R_1}V_{in} & -\frac{1}{R} \\ 0 & sC \end{vmatrix}}{\begin{vmatrix} \frac{1}{QR}+sC & -\frac{1}{R} \\ \frac{1}{R} & sC \end{vmatrix}} = \frac{-\frac{1}{R_1} sC\, V_{in}}{sC(\frac{1}{QR}+sC)+\frac{1}{R^2}} = \frac{-\frac{R}{R_1} sRCV_{in}}{1+s\frac{RC}{Q}+s^2R^2C^2}$$

$V_2(s) =$ ______

2p|z BPF 2p LPF

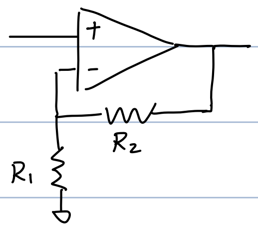
Opamp (bw)

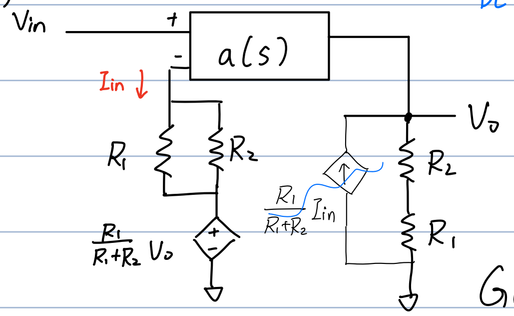


Single pole $A(s) = \frac{A_0}{1+\frac{s}{w_u/A_0}}$ → corner at $\frac{w_u}{A_0}$; unity at w_u

w_u : GBW (if 1st order up to w_u)

E.ni





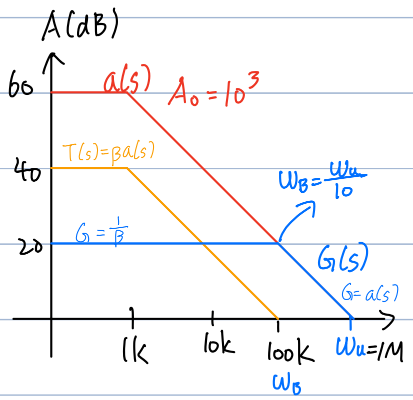
$G(s) = \frac{a(s)}{1+\beta a(s)}$

$= \frac{A_0}{1+s\frac{A_0}{w_u}+\beta A_0}$

$= \frac{A_0}{(1+\beta A_0)+s\frac{A_0}{w_u}}$

$= \frac{A_0}{1+\beta A_0} \cdot \frac{1}{1+\frac{s}{w_B}}$ $\left(w_B = w_u \frac{\cancel{1+\beta A_0}}{A_0}\right)$

$\approx \frac{1}{\beta} \cdot \frac{1}{1+s/w_B}$, $w_B = \frac{w_u}{1/\beta}$



$\beta = \frac{1}{10}$, $T(s) = \beta a(s)$

$G = \frac{1}{\beta} \frac{T(s)}{1+T(s)}$

$T(s) >> 1$, $G = \frac{1}{\beta}$ → BW↑

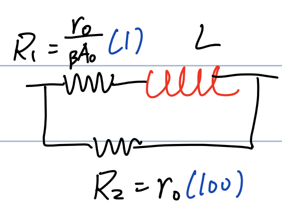
$T(s) < 1$, $G = a(s)$ (rolloff)

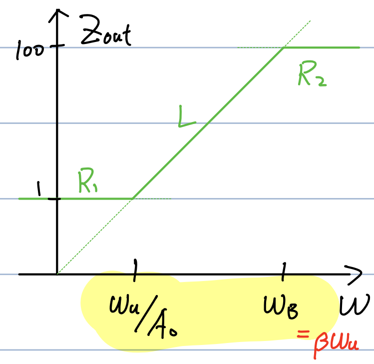
$Z_{out}(s) = \frac{R_{out}}{1+T(s)} = \frac{r_o \,\cancel{\|(R_1+R_2)}}{1+T(s)}$

Model w/ L (Z↑ as w↑) in sr.

E. $R_{out} = 100\Omega$: Z_{out} : (1Ω ↗ 100Ω)

$L = \frac{r_o}{w_B} = \frac{r_o}{\beta w_u} \approx 160\,\mu H$





If Z_L is cap → resonate :(

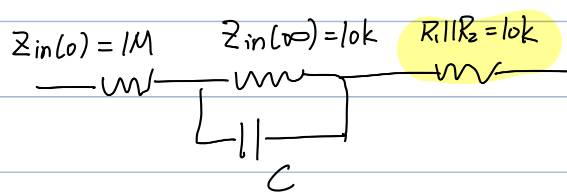
10k · 100

$Z_{in}(s) = (R_1 \| R_2) + r_{in}(1+a(s)\beta)$ only r_{in}!

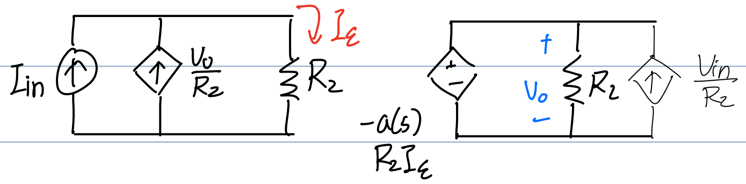
E. $R_{in} = 10k$, $R_1 \| R_2 = 10k$

$Z_{in}(s) = 10k + 10k(1+a(s)\beta)$

$Z_{in}(0) = 1\,M\Omega$, $Z_{in}(\infty) = 20\,k$ $Z_c = \frac{1}{jwC}$ $C \approx 0.16\,nF$



E. IA (sh-sh)



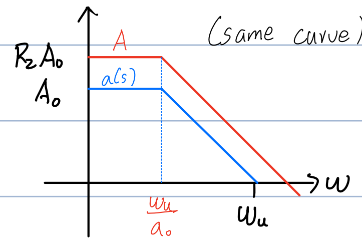
$A(s) = \frac{V_o}{I_\varepsilon} = -R_2\, a(s)$

$\beta = -\frac{1}{R_2}$

$T = \beta A = a(s)$

$G = \frac{A}{1+\beta A}$

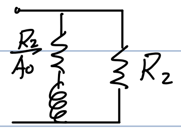
If R_s is present in src → G − BW tradeoff



Z_{in} (assume $r_{in} = \infty$) $= \frac{R_{in}}{1+T(s)}$ (Miller effect)

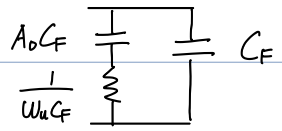
$R_{in} = R_2$

$L_{eq} = \frac{R_2}{\omega_u}$



$Z = j\omega L$

If $Z_f = \frac{1}{sC_F}$

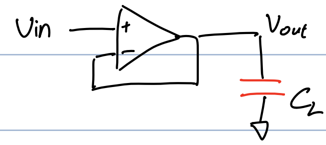


$a(\omega_u) = 1$

Z_{out}??

Stability

E. unity buf. $\beta = 1$, $\omega_B = 1MHz$

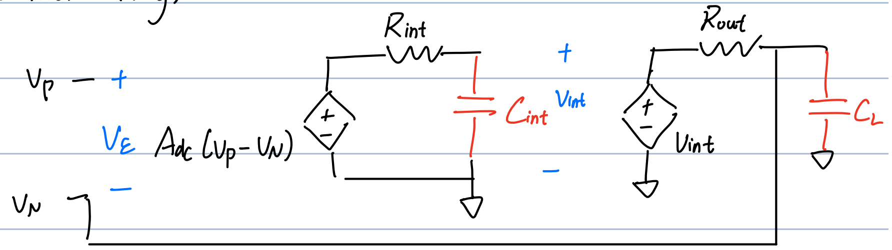


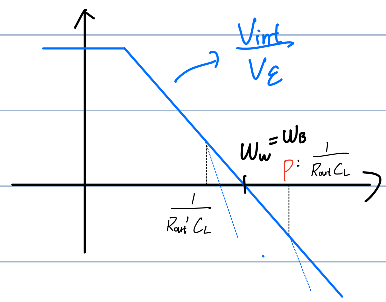
If $C_{out} \uparrow \rightarrow$ ringing! More severe if $C_L \uparrow$

If $\omega_B \uparrow \rightarrow$ ringing $\uparrow$ (faster, but more ring)

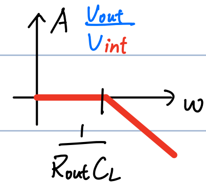
Two poles: C_{int} and C_L

two stages to isolate



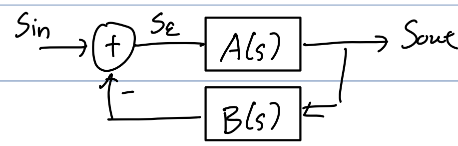


additional pole at $\omega = \frac{1}{R_{out} C_L}$



$G(s) = \frac{A(s)}{1+\beta(s)A(s)}$ $\approx \begin{cases} \frac{1}{\beta(s)} & \text{for } A\beta \gg 1 \\ A(s) & \text{for } A\beta \ll 1 \end{cases}$

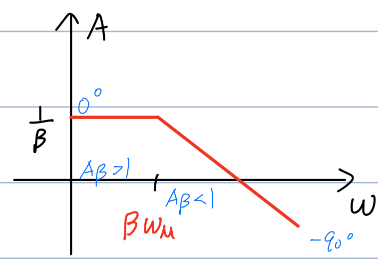
(fine if $A\beta \neq -1$)



Let $A(s) = \frac{N(s)}{D(s)}$ $\begin{matrix} z \\ p \end{matrix}$ $\rightarrow G(s) = \frac{N(s)}{D(s)+\beta N(s)}$ (assume $\beta(s) = \beta$)

zero of $A(s) \rightarrow z$ of $G(s)$

pole of $A(s) \rightarrow$?? matters for stability



1st order

E. $\beta(s) = \beta$, $A(s) = \frac{\omega_u}{s} \rightarrow G(s) = \frac{\omega_u}{s + \beta\omega_u}$

$p = -\beta\omega_u < 0 \rightarrow$ stable

Second order E. $A(s) = \frac{A_0}{(\frac{s}{\omega_{p1}}+1)(\frac{s}{\omega_{p2}}+1)}$

$G(s) = \frac{A(s)}{1+\beta A(s)} = \frac{\frac{A_0\omega_{p1}\omega_{p2}}{(s+\omega_{p1})(s+\omega_{p2})}}{\frac{(s+\omega_{p1})(s+\omega_{p2}) + \beta A_0 \omega_{p1}\omega_{p2}}{(s+\omega_{p1})(s+\omega_{p2})}}$

$\therefore G(s) = \frac{A_0 \omega_{p1}\omega_{p2}}{s^2 + (\omega_{p1}+\omega_{p2})s + (1+A_0\beta)\omega_{p1}\omega_{p2}}$ Let $\omega_0^2 = (1+A_0\beta)\omega_{p1}\omega_{p2}$

$\equiv \frac{A_0}{s^2 + (\frac{\omega_0}{Q})s + \omega_0^2}$ $Q = \frac{\sqrt{(1+A_0\beta)\omega_{p1}\omega_{p2}}}{\cancel{\omega_{p1}} + \omega_{p2}}$

If $\omega_{p2} \gg \omega_{p1}$, $A\beta \gg 1$, $Q \approx \sqrt{\frac{A_0\beta\omega_{p1}}{\omega_{p2}}} \equiv \sqrt{\frac{\omega_u}{\omega_{p2}}}$

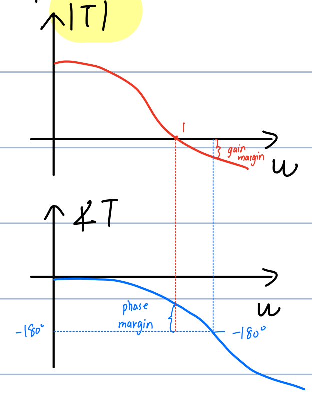
We typically want $Q < 1$, so $\omega_{p2} > \omega_u$ (worst case unity)

$Q = 0.5$: critically damped $\omega_u = 0.25\,\omega_{p2}$

under

Freq domain criteria stability given $\beta(s)A(s)$

$G(s)=\frac{A(s)}{1+\beta(s)A(s)}$ → p at $A(s)\beta(s)=-1$, stable if all poles in LHP



1. ω_π where $\measuredangle[A(j\omega_\pi)B(j\omega_\pi)]=-180^\circ \Rightarrow |A(j\omega_\pi)B(j\omega_\pi)|<1$

   At phase crossover, $|T|<1$ (gain margin $\frac{1}{|T|}>1$)

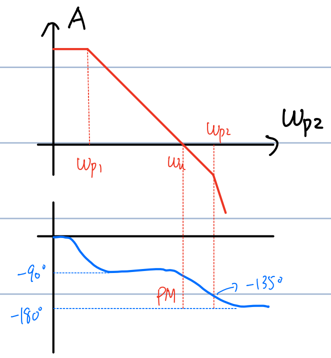
2. ω_u where $|A(j\omega_u)B(j\omega_u)|=1 \Rightarrow \measuredangle(\text{———})>-180^\circ$

   At gain crossover, $\measuredangle T>-180^\circ$ (phase margin $180^\circ+\measuredangle T>0$)

if too close to margin, risk unstable

ω_π and ω_u need to be uniquely defined. Else need Nyquist test

E.



If add p_3, it will drag down $\measuredangle$ → PM ↓

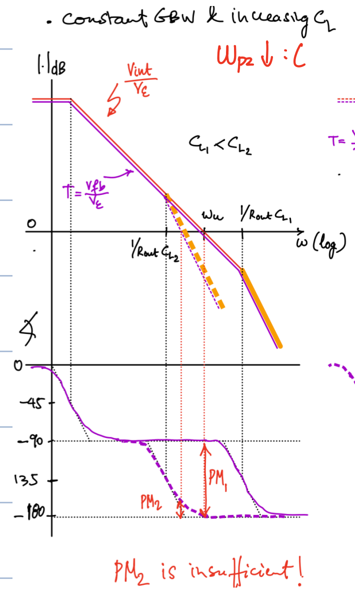
If β ↓, $T(s)$ ↓, crossover at lower ω → PM ↑

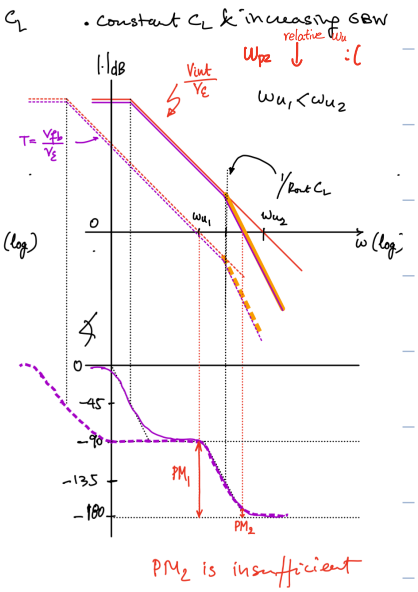
C_L ↑ pole $\frac{1}{R_{out}C}$ ↓ (both A and $\measuredangle$) → PM ↓

Aim for PM > 45°, 60° → no ringing, critically damped

Need $\omega_u = A_0\omega_{p1} < \omega_{p2} < \omega_{p3}$ (higher p no interfere)

if $\beta<1$, $T(s)=\beta A(s)$ ↓ → ω_B ↓ → pm ↑

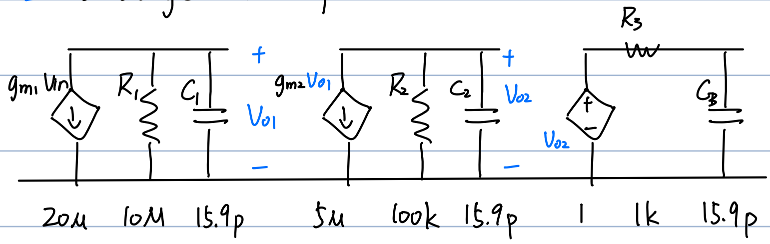




Compensation $\quad$ change $\beta(s)$ if opamp given

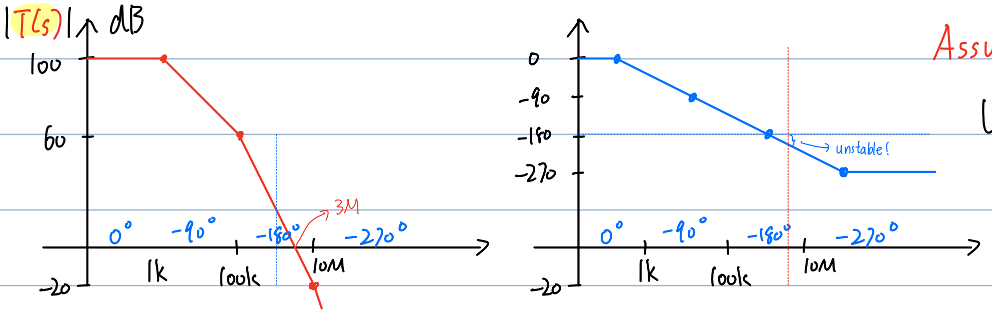
change $A(s)$ if designing opamp

E. 3-stage w/ 3 poles



DC $\quad A_1 = -200 \quad A_2 = -500 \quad A_3 = 1 \quad A = 100k$ (100 dB)

p (Hz) $\quad -1kHz \quad -100k \quad -10M \quad BW = 1kHz$



Assume $\beta = 1$ (worst case)

Unstable :(

Sln. (can't change cap) Need $\omega_u = \omega_{p2}$. → 45° pm

1. E Reduce gain @ p_2 to 0dB → reduce A by 60 dB

:(Less benefit from feedback E. accuracy, Z_{in} / Z_{out}

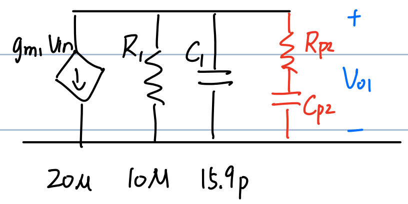
2. E. add Cap. (dominant pole compensation) Make $C_1 = 15.9\,nF \rightarrow p_1 = 1\,Hz$ $\quad$ (pull p_1 low)

$\omega_u = \omega_{p2} = 100kHz \rightarrow \omega_{p1} = \frac{100k}{10^{100/20dB}} = 1Hz$

pm ↑, bw ↓ :(

* Moving p_2 won't help. Need all other poles beyond $\omega_u = A_0\,\omega_d$

3. p/z compensation Add z to cancel p_2. (R dom. below p ; C above).



$\omega_{p1} = -\frac{1}{R_1(C_1+C_{pz})}$ $R_{pz} \approx 0$ Assume $R_{pz} \ll R_1$

$\omega_z = -\frac{1}{R_{pz}C_{pz}}$ $C_{pz} \gg C_1$

$\omega_{p4} = -\frac{1}{R_{pz}C_1}$ $R_1 \approx \infty$ $\omega_{p1} \ll \omega_{pz}$

Also add zeroes to cancel poles

Problem: if p/z misalign → :(more overshoot, -

E. $R_{pz} = 500k$, $C_{pz} = 500p$

$\omega_4 = GBW = \omega_1 a_0 \rightarrow \frac{1}{R_{pz}C_1} = a_0 \frac{1}{R_1(C_{pz}+\cancel{C_1})}$

$\omega_z = \frac{1}{R_{pz}C_{pz}} = \omega_2$

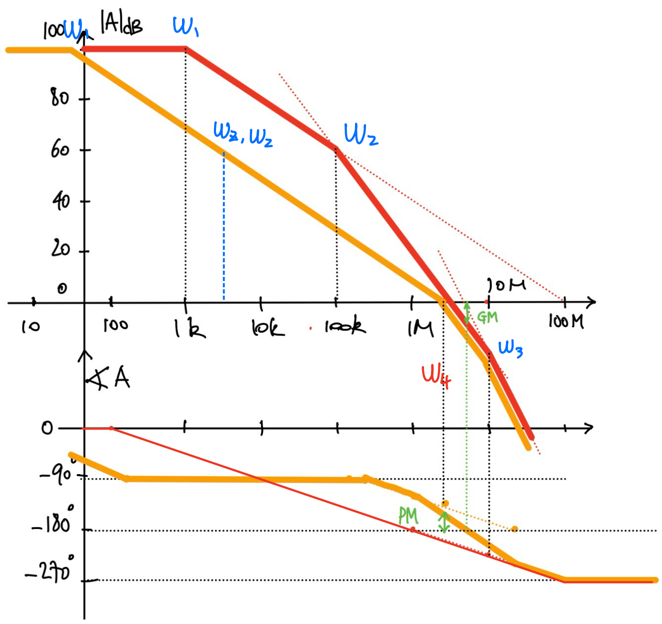
$\rightarrow C_{pz} \doteq \sqrt{\frac{a_0 C_1}{R_1 \omega_2}} = 500p$

$R_{pz} = \frac{1}{\omega_2 C_{pz}} = 3.2k$

$\omega_1 \doteq 31$ Hz

$\omega_4 \doteq 31$ MHz → 45° pm

idea: cancel ω_2 w/ ω_z. Replace w/ ω_4 @ unity



Caveat if p/z misaligned → problem

	p_1 (rad/s)	p_2	GBW	$p_3 \cdots$	A_0'
Reduce A_0	$\frac{1}{R_1C_1}$	$\frac{1}{R_2C_2}$	$p_2 = \frac{1}{R_2C_2}$		$A_0 \frac{\omega_1}{\omega_2}$
dom. pole	$\frac{1}{R_1C_1'}$ ↑	$\frac{1}{R_2C_2}$			
p/z comp.	$\frac{1}{R_1(C_1+C_{pz})}$	$\frac{1}{R_{pz}C_{pz}} = \omega_2$	$\frac{1}{R_{pz}C_1} = \omega_4$		
Miller	↑	↓	$\frac{g_{m1}}{C_c}$		

Miller (sh- sh feedback)

if comp put across gain element, $z \uparrow$

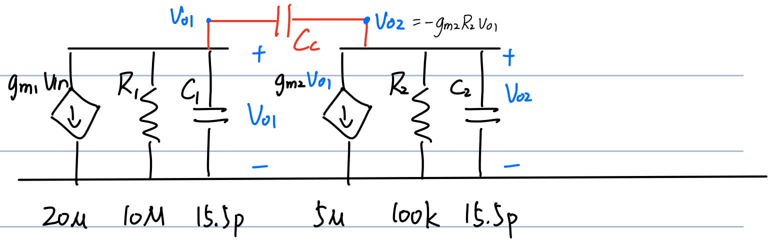
$V_{Cc} = V_{o1} - -g_{m2}R_2 V_{o1} = (1+\overbrace{g_{m2}R_2}^{A_2})V_{o1}$

$\omega_1 = \dfrac{1}{R_1(\cancel{C_1}+C_{eq})} = \dfrac{1}{R_1 A_2 C_c}$ ($\downarrow$)

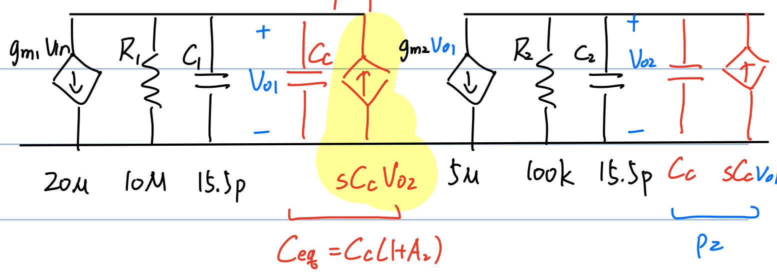
$GBW = \dfrac{1}{R_1 \cancel{A_2} C_c}(g_{m1}R_1\cancel{A_2}) = \dfrac{g_{m1}}{C_c}$

$g_{m2eq} = g_{m2} - sC_c$

$\hookrightarrow$ RHP z @ $\omega_{z2} = \dfrac{g_{m2}}{C_c}$



$\downarrow$ y-param (sh-sh)



$$\frac{V_{o2}(s)}{V_{in}(s)} = \frac{g_{m1}R_1 g_{m2}R_2 (\overbrace{1-\frac{s}{g_{m2}/C_c}}^{z})}{1+s((C_1+C_c(1+g_{m2}R_2))R_1)+s^2(R_1R_2((C_2+C_c)C_1+C_2C_c))}$$

$$= \frac{N(s)}{1+as+bs^2} = \frac{N(s)}{(1+\frac{s}{\omega_1})(1+\frac{s}{\omega_2})}$$

if $\omega_1 \ll \omega_2$ $\quad \dfrac{1}{\omega_1} = R_1(C_c(1+g_{m2}R_2)+C_1)+R_2(C_c+C_2)$

$$\omega_2 = \frac{\cancel{R_1}(C_c(\cancel{1+}g_{m2}\cancel{R_2}))}{\cancel{R_1R_2}((C_2+C_c)C_1+C_2C_c)} \approx \frac{g_{m2}C_c}{C_1C_2+C_1C_c+C_2C_c}$$

if $C_2 \gg C_c$, $C_2 \gg C_1$

$\omega_1' \approx \omega_1 \dfrac{C_1}{g_{m2}R_2C_c}$ $\downarrow$

$\omega_2' \approx g_{m2}R_2\,\omega_2$ $\quad (C_2, C_c \gg C_1)$ $\uparrow$

	$C_c = 0$		C_c large
ω_1	$\dfrac{1}{R_1C_1}$	$\gg$	$\dfrac{1}{R_1(\cancel{C_1}+g_{m2}R_2C_c)}$
ω_2	$\dfrac{1}{R_2C_2}$	$\ll$	$\dfrac{g_{m2}}{\cancel{C_1}+C_2+\frac{C_1C_2}{\cancel{C_c}}}$
RHP z	$\times$		$\dfrac{g_{m2}}{C_c}$

p_2 up !!! (pole splitting) $\longrightarrow$ pm $\uparrow$

additional z LHP z $0^\circ \rightarrow +90^\circ$ help pm leading 90

RHP z $0^\circ \rightarrow -90^\circ$? hurt pm lagging 90

Need push z high , or make it LHP

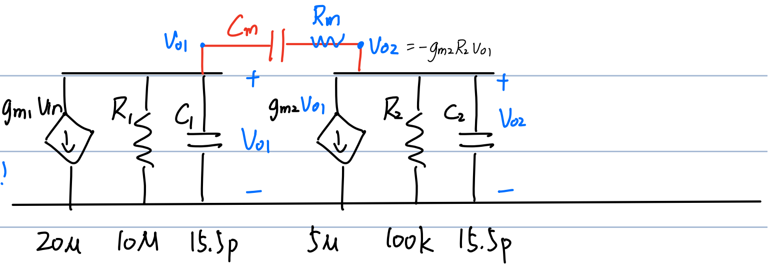
If add large C directly , slow RC

but Miller, RC $\downarrow$,

If add R_c, less overshoot /oscillation (improved PM)

RHP zero compensation

Add $C_M - R_M$, avoid C_c feedfwd ($sC_c V_{o1}$)

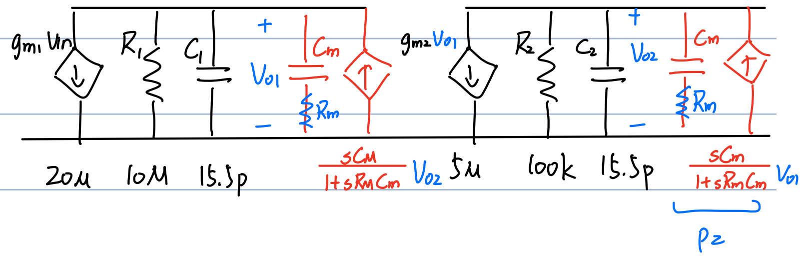


$$g_{m2,eq} = g_{m2} - \frac{sC_M}{1+sR_MC_M} = g_{m2}\frac{1+sC_M(R_M - 1/g_{m2})}{1+sR_MC_M} \rightarrow z!$$

If $R_M > \frac{1}{g_{m2}} \rightarrow$ no RHP z

(LHP helps)

↓ y-param (sh-sh)



V_{cm} is a new state var. $\rightarrow$ 3p

if $g_{m1}R_1,\ g_{m2}R_2 >> 1,\ C_M >> C_1,\ C_2 >> C_1,$

$$\left.\begin{array}{l} \omega_{p1} \approx \dfrac{1}{R_1 g_{m2} R_2 C_m} \\ \omega_{p2} \approx \dfrac{g_{m2}C_m}{C_1C_2 + C_1C_m + C_2C_m} \end{array}\right\} \text{same}$$

$$\omega_{p3} \approx \frac{1}{R_mC_1} \longrightarrow \text{new}$$