$\sin n\pi = 0$
$1 - \cos n\pi = 2$ for odd n

$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$
$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$

$\sin x \sin y = {}^1\!/\!_2[\cos(x-y) - \cos(x+y)]$
$\cos x \cos y = {}^1\!/\!_2[\cos(x-y) + \cos(x+y)]$
$\sin x \cos y = {}^1\!/\!_2[\sin(x-y) + \sin(x+y)]$

$C\,\mathrm{c}\,(\omega_0 t + \theta) = C\,\mathrm{c}\,(\theta)\,\mathrm{c}\,(\omega_0 t) - C\mathrm{s}\,(\theta)\mathrm{s}\,(..)$

$\theta = \tan^{-1}(-b/a)$, $\pm\pi$ when $a < 0$
$\sin t = \cos(t - \pi/2)$

$\cos x = \frac{1}{2}\left[e^{jx} + e^{-jx}\right]$
$\sin x = \frac{1}{2j}\left[e^{jx} - e^{-jx}\right]$
$e^{j\omega t} = \cos(\omega t) + j\sin(\omega t)$

$z^* = a - jb = re^{-j\theta}$
$u^* v^* = (uv)^*$

$\angle z = \tan^{-1}(b/a)$, $\pm\pi$ in Q2 and Q3

$z^{\frac{1}{n}} = r^{\frac{1}{n}} e^{j\frac{\theta + 2\pi m}{n}}$

$\int \cos^2 at\, dt = \frac{t}{2} + \frac{\sin 2at}{4a}$

$\int t\cos at\, dt = \frac{1}{a^2}(\cos at + at\sin at)$
$\int t^2 \mathrm{c}\, at\, dt = \frac{1}{a^3}(2at\mathrm{c}\, at - 2\mathrm{s}\, at + a^2t^2\mathrm{s}\, at)$

$\int te^{at}\, dt = {}^1\!/\!_{a^2}\, e^{at}(at - 1)$
$\int t^2 e^{at}\, dt = {}^1\!/\!_{a^3}\, e^{at}(a^2t^2 - 2at + 2)$

$\int e^{at}\, \mathrm{c}\, bt\, dt = \frac{1}{a^2+b^2}\, e^{at}(a\cos bt + b\sin bt)$

$\int \frac{1}{x^2+a^2}\, dx = \frac{1}{a}\tan^{-1}\frac{x}{a}$

$\mathcal{E}_f = \int_{-\infty}^{\infty} |f(t)|^2 dt$ (complex);
$P_f = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} |f(t)|^2 dt$;
rms power $= \sqrt{P_f}$

Cont; analog; periodic (extension);
(non/anti)causal; energy/power (both);
deterministic/stochastic (carries info)

$\int f(t)\cdot\delta(t - t_0)dt = f(t_0)$ (f cont at t_0)

out–in $f(2x-6)$: shift by 6, scale by 2;
$f(2(x-6))$: scale by 2, shift by 6

$f_e(t) = {}^1\!/\!_2[f(t) + f(-t)]$;
$f_o(t) = {}^1\!/\!_2[f(t) - f(-t)]$

L: $\mathcal{T}[kf_1(t) + f_2(t)] = ky_1(t) + y_2(t)$.
$\mathcal{T}$: $\sum_{k=0} a_k D^k\, y(t) = \sum_{l=0} b_l D^l\, f(t)$,
L if a_k, b_l are not functions of $y(t), f(t)$
E. $\sin\dot{y}(t) + t^2 y(t) = (t+3)f(t)$

TI: $\mathcal{T}[f(t-\tau)] = y(t-\tau)$.
a_k, b_l indep of t. (const coeff)
Let $g(t) \equiv f(t-\tau)$, find $z(t) = \mathcal{T}[g(t)]$

Causal: $y(t)$ dep only on $f(\tau)$, $\tau < t$.
Just compare t and τ.

Ins/dyn: y only dep f at present (no $\int$)

Invertible: given $y(t)$, we can know $f(t)$

$c(t) \equiv \int_{-\infty}^{\infty} f(\tau)g(t-\tau)d\tau$

$f * g = g * f$

$f * (g + h) = f * h + g * h$

$f * (g * h) = (f * g) * h$

Pf: $f * (g * h) = f * (h * g) =$
$\int f(\tau_1)\int h(\tau_2)\, g(t - \tau_1 - \tau_2)\, d\tau_2\, d\tau_1$

$f(t - T_1) * g(t - T_2) = c(t - T_1 - T_2)$

$f(at) * g(at) = |{}^1\!/\!_a|\, c(at)$ (even/odd)

$f^{(m)}(t) * g^{(n)}(t) = c^{(m+n)}(t)$

Graph: shift LEFT by t, and reflect.
Every τ replaced by $t - \tau$, reverted

$f(t) * \delta(t - T) = f(t - T)$

$u(t) * u(t) = t\, u(t)$

$e^{at}\, u(t) *\, u(t) = \frac{1 - e^{at}}{-a}\, u(t)$

$e^{at}\, u(t) * e^{bt}\, u(t) = \frac{e^{at} - e^{bt}}{a - b}\, u(t)$
($te^{at}\, u(t)$ for $a = b$)

$e^{at}\, u(t) * e^{bt}\, u(-t) = \frac{e^{at}\, u(t) + e^{bt}\, u(-t)}{b - a}$

$te^{at}\, u(t) * e^{at}\, u(t) = {}^1\!/\!_2\, t^2 e^{at}\, u(t)$

$t^m\, u(t) * t^n\, u(t) = \frac{m!n!}{(m+n+1)!}\, t^{m+n+1}\, u(t)$

Don't forget $[u(t + T_1) - u(t - T_2)]$ term

$Q(D)y(t) = P(D)f(t)$, typically $\int f$
Assume causal input $f(t)u(t)$

$y_{zs}(t) = f(t) * h(t)$ from input
$y_{zs}(0^-) = 0$, $y_{zs}(0^+) \neq 0$

Let $h(t) = \mathcal{T}[\delta(t)]$ (impulse response)
$y_{zs}(t) = \mathcal{T}[f(t)] = \mathcal{T}[f(t) * \delta(t)] =$
$\mathcal{T}[\lim \sum f(n\Delta\tau)\delta(t - n\Delta\tau)\Delta\tau] =$
$\lim \sum f(n\Delta\tau)h(t - n\Delta\tau)\Delta\tau = f * h$

$y_{zi}(t)$ from ini, $f(t) = 0$, $Qy_{zi}(t) = 0$;
$y_{zi}(0^-) = y_{zi}(0^+)$

$\mathcal{E}_e = \int_{t_1}^{t_2} [e(t)]^2 dt = \int_{t_1}^{t_2} f^2(t) dt$
$-2\sum c_i \int f(t)x_i(t)dt + \int(\sum c_i x_i(t))^2 dt$
$= \mathcal{E}_f - 2\sum\langle f, x_i\rangle + (\sum c_i^2 \int x_i(t)^2 dt +$
$\sum_{i\neq j} c_i c_j \int_{t1}^{t2} x_i(t)x_j(t)dt)$

$\frac{\partial\mathcal{E}_e}{\partial c_i} = 0 = -2\langle f(t), x_i(t)\rangle + 2\mathcal{E}_i c_i$
$\mathcal{E}_e^{\min} = \mathcal{E}_f - \sum_{i=1}^{N} c_i^2 \mathcal{E}_i$

$\mathcal{E}_e^{\min} = 0$, Parseval's thm

For ortho, $E_z = E_x + E_y$

$|u + v|^2 = |u|^2 + |v|^2 + u^*v + v^*u$

$\langle x(t), y(t)\rangle = \int_{t1}^{t2} x(t)y(t)^*\, dt$

$a_0 = \frac{1}{T_0}\int_{T_0} f(t)\, dt$
$a_n = \frac{2}{T_0}\int_{T_0} f(t)\cos(n\omega_0 t)\, dt$
$b_n = \frac{2}{T_0}\int_{T_0} f(t)\sin(n\omega_0 t)\, dt$
Energy: T_0 for $n = 0$; $T_0/2$ else

Half-w sym: $f(t - {}^{T_0}\!/\!_2) = -f(t)$
$a_{n_{\mathrm{odd}}} = \frac{4}{T_0}\int_0^{T_0/2} f(t)\cos(n\omega_0 t)\, dt$

$f(t) = \sum_{n=-\infty}^{\infty} F_n e^{jn\omega_0 t}$
$F_n = \frac{1}{T_0}\int_{T_0} f(t)e^{-jn\omega_0 t}\, dt$

$C_n\cos(n\omega_0 t + \theta_n) =$
${}^{C_n}\!/\!_2\, (e^{j(n\omega_0 t + \theta_n)} + e^{-j(n\omega_0 t + \theta_n)}) =$
$(\frac{C_n}{2}e^{j\theta_n})e^{jn\omega_0 t} + (\frac{C_n}{2}e^{-j\theta_n})e^{-jn\omega_0 t}$

$F_n = \frac{C_n}{2}\, e^{j\theta_n} = \frac{1}{2}(a_n - jb_n) = |F_n|e^{j\angle F_n}$
$F_{-n} = \frac{C_n}{2}\, e^{-j\theta_n}$

W: finite $\int$, fin a, b;

S: fin m/m/discont over T_0: converge

Time shift: $f(t - t_0) \leftrightarrow F_n e^{-jn(\omega_0 t_0)}$,
$|F_n|$ same, $\angle F_n$ shifted by $-(\omega_0 t_0)n$

Reversal: $f(-t) \leftrightarrow F_{-n}$

Scal: $T = {}^{T_0}\!/\!_a$, $\omega = a\omega_0$

Multip (same T_0): $f(t)g(t) \leftrightarrow F_n * G_n$
$\frac{1}{T_0}\int_{T_0} f(t)g(t)e^{jn\omega_0 t}\, dt =$
$\frac{1}{T_0}\int(\sum F_m e^{jm\omega_0 t})(\sum G_k e^{jk\omega_0 t})e^{-jn\omega_0 t}\, dt$
$= \sum_m \sum_k F_m G_k \frac{1}{T_0}\int_{T_0} e^{j(m+k-n)\omega_0 t}\, dt$
$= \sum_m \sum_k F_m G_k \langle e^{j(m+k)\omega_0 t}, e^{jn\omega_0 t}\rangle$
$= \sum_{k=-\infty}^{\infty} G_k F_{n-k}$

Conjugation: $f(t)^* = F_{-n}^*$

f real $\to |F|$ even, $\angle F$ odd
f real, even $\to F$ re, e; $F_{-n} = F_n = F_n^*$
f re, od $\to F$ im, o; $-F_{-n} = F_n = -F_n^*$

$f_e(t) \leftrightarrow \mathrm{Re}\{F_n\}$
$f_o(t) \leftrightarrow j\,\mathrm{Im}\{F_n\}$

Square ($A = 1$, $T = 2\pi$, $\omega = 1$)
$\frac{4}{\pi}(\cos t - \frac{1}{3}\cos 3t + \frac{1}{5}\cos 5t - ...)$
$\frac{4}{\pi}(\sin t + \frac{1}{3}\sin 3t + \frac{1}{5}\sin 5t + ...)$

Triangle:
$\frac{8}{\pi^2}(\sin t - \frac{1}{9}\sin 3t + \frac{1}{25}\sin 5t - ...)$
$\frac{8}{\pi^2}(\cos t + \frac{1}{9}\cos 3t + \frac{1}{25}\cos 5t + ...)$

Sawtooth:
$\frac{2}{\pi}(\sin t - \frac{1}{2}\sin 2t + \frac{1}{3}\sin 3t - ...)$
$\frac{2}{\pi}(-\sin t - \frac{1}{2}\sin 2t - \frac{1}{3}\sin 3t - ...)$

δ train: $\delta_{T_0}(t) = \sum_{n=-\infty}^{\infty}\delta(t - nT_0)$
$\frac{1}{T_0}\sum_{n=-\infty}^{\infty} e^{jn\omega_0 t}$