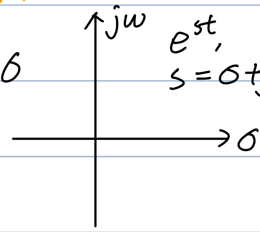
FT, f needs to be energy. $\int_{-\infty}^{\infty} |f(t)|^2 dt < \infty$! → Laplace transform

e^{st}, $s = \sigma + j\omega$ (complex). For FT, $s = j\omega$. Extend by adding real part: σ



e^{st}, $s = \sigma + j\omega$

1. Unilateral LT

$F(s) = \mathcal{L}\{f(t)\}$

$= \int_{0^-}^{\infty} f(t)\, e^{-st}\, dt$, $s = \sigma + j\omega$

$= \int_{0^-}^{\infty} [f(t)\, e^{-\sigma t}]\, e^{-j\omega t}\, dt$ Hope $\cdot e^{-\sigma t}$ con.

meaningful for causal $f(t)$ only

For causal, $F(\omega) = F(s)|_{s=j\omega}$

Converges if $\int_{0^-}^{\infty} |f(t)\, e^{-\sigma t}|\, dt < \infty$.

Let σ_0 be smallest such σ. Region of convergence (ROC): $Re\{s\} > \sigma_0$

E. $\mathcal{L}\{\delta(t)\} = \int_{0^-}^{\infty} \delta(t)\, e^{-st}\, dt = 1$ ROC: any s

E. $\mathcal{L}\{u(t)\} = \int_{0^-}^{\infty} \cancel{u(t)}\, e^{-st}\, dt = \frac{1}{s}$ $Re\{s\} > 0$

E. $\mathcal{L}\{e^{at} u(t)\} = \int_{0^-}^{\infty} e^{at}\, e^{-st}\, dt = \frac{1}{s-a}$ $Re\{s\} > Re\{a\}$

Prop.	ROC
1. Linearity	$\cap$!
2. Time shift $f(t-t_0)\, u(t-t_0) \leftrightarrow F(s)\, e^{-st_0}$ (for causality)	
Pf. $t - t_0 = \tau$, $\cdots = \int_{\cancel{-t_0}\ 0}^{\infty} f(\tau)\, u(\tau)\, e^{-s\tau}\, e^{-st_0}\, d\tau = \cdots$	same
3. s-shift $f(t)\, e^{s_0 t} \leftrightarrow F(s - s_0)$	$Re(s) > \sigma_0 + Re(s_0)$
E. $\mathcal{L}[\cos \omega_0 t\, u(t)] = \mathcal{L}[\frac{1}{2}(e^{j\omega_0 t} + e^{-j\omega_0 t})\, u(t)]$	
$= \frac{1}{2} \mathcal{L}[u(t)\, e^{j\omega_0 t} + u(t)\, e^{-j\omega_0 t}]$	
$= \frac{1}{2}\left[\frac{1}{s - j\omega_0} + \frac{1}{s + j\omega_0}\right]$ → − ? + ?	
$= \frac{s}{s^2 + \omega_0^2}$	$Re(s) > 0$
4. Scaling $a > 0$ (causal), $f(at) \leftrightarrow \frac{1}{a} F(\frac{s}{a})$	$Re(s) > a\sigma_0$
5. Convolution $f_1 * f_2 \leftrightarrow F_1 \cdot F_2$	
(other way also works but too hard :( )	$\cap$

6. Time diff. $\dot f(t) \leftrightarrow sF(s) - \overset{\text{init cond.}}{f(0^-)}$

Pf. $\int_{0^-}^{\infty} \dot f(t) e^{-st}\, dt \overset{\text{parts}}{=} \left[e^{-st} f(t)\right]_{0^-}^{\infty} + s \int_{0^-}^{\infty} f(t) e^{-st}\, dt$

$= -f(0^-) + sF(s)$ $Re(s) > \max(\sigma_0, 0)$

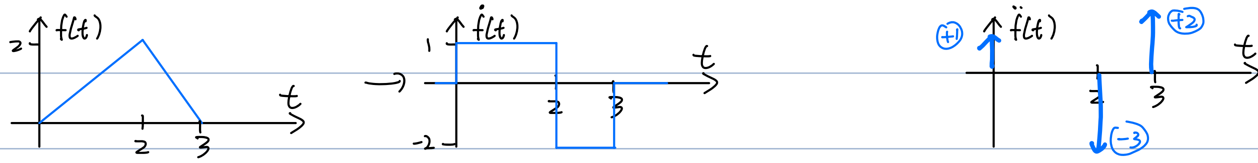
DFQ → algebra!

$\ddot f(t) \leftrightarrow s\,\mathcal{L}\{\dot f(t)\} - \dot f(0^-)$

$= s^2 F(s) - s f(0^-) - \dot f(0^-)$

$f^{(n)}(t) \leftrightarrow s^n F(s) + \sum_{i=0}^{n-1} s^{n-1-i} f^{(i)}(0^-)$

E.



$\mathcal{L}[\ddot f(t)] = \mathcal{L}[\delta(t) - 3\delta(t-2) + 2\delta(t+3)]$

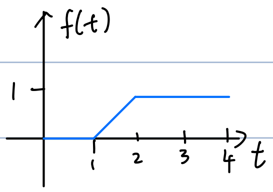
$s^2 F(s) - \cancel{s f(0)} - \cancel{\dot f(0^-)} = 1 - 3e^{-2s} + 2e^{-3s}$

$F(s) = \frac{1}{s^2}\left(1 - 3e^{-2s} + 2e^{-3s}\right)$

7. freq-diff $-t f(t) \leftrightarrow \frac{dF(s)}{ds}$ Pf. $\frac{dF(s)}{ds} = \int_{0^-}^{\infty} -t f(t) e^{-st}\, dt$ ——

E. $u(t) \leftrightarrow \frac{1}{s}$ unit ramp $t\,u(t) \leftrightarrow \frac{1}{s^2}$

E.



$f(t) = (t-1)[u(t-1) - u(t-2)] + [u(t-2) - u(t-4)]$

$= (t-1)u(t-1) - (t-2)u(t-2) - u(t-4)$

$F(s) = \frac{1}{s^2} e^{-s} - \frac{1}{s^2} e^{-2s} - \frac{1}{s} e^{-4s}$

8. Time $\int$ $\int_{0^-}^{t} f(\tau)\, d\tau \leftrightarrow \frac{1}{s} F(s)$

Pf. Let $y(t) \equiv \int_{0^-}^{t} f(\tau)\, d\tau$

$\frac{dy(t)}{dt} = f(t)$, $y(0^-) = 0$

$sY(s) - \cancel{y(0^-)} = F(s)$

9. Init value thm. $f(0^+) = \lim_{s\to\infty} sF(s)$ if exists

Pf. $\mathcal{L}[\dot f(t)] = \int_{0^-}^{\infty} \dot f(t)\, e^{-st}\, dt$

$sF(s) - f(0^-) = \int_{0^-}^{0^+} \dot f(t) \overset{1}{\cancel{e^{-st}}}\, dt + \int_{0^+}^{\infty} \dot f(t) \overset{\lim_{s\to\infty} = 0}{\cancel{e^{-st}}}\, dt$

$\lim_{s\to\infty} sF(s) - \cancel{f(0^-)} = f(0^+) - \cancel{f(0^-)}$

10. Final value thm. $\lim_{t\to\infty} f(t) = \lim_{s\to 0} sF(s)$ if exists

Pf. $sF(s) - f(0^-) = \int_{0^-}^{\infty} \dot f(t)\, e^{-st}\, dt$

$\lim_{s\to 0} sF(s) - f(0^-) = \lim_{s\to 0} \int_{0^-}^{\infty} \dot f(t)\, \cancel{e^{-st}}^{1}\, dt$

$\lim_{s\to 0} sF(s) - \cancel{f(0^-)} = \lim_{t\to\infty} f(t) - \cancel{f(0^-)}$

E. $Y_s = \frac{10(2s+3)}{s(s^2+2s+5)}$ $\quad y(0^+) = 0$

$y(\infty) = \frac{10\cdot 3}{5} = 6$

$\mathcal{L}^{-1}$ for rational (poly/poly) fucns. (const. coeff. ODEs)

$F(s) = \frac{b_m s^m + \cdots + b_1 s + b_0}{s^n + \cdots + a_1 s + a_0}$ $\begin{matrix} m\text{-order} \\ n\text{-order} \end{matrix} \equiv \frac{P(s)}{Q(s)}$

Proper if $m < n$. Else improper $=$ poly^{m-n} + proper

E. $F(s) = \frac{2s^3+9s^2+11s+2}{s^2+4s+3} \overset{\text{long}\int}{=} (2s+1) + \frac{s-1}{s^2+4s+3}$

$f(t) = \underbrace{2\dot\delta(t) + \delta(t)}_{\dot\delta \text{ uninteresting}} + \mathcal{L}^{-1}\left[\frac{s-1}{s^2+4s+3}\right]$

Partial frac

1. Q has unrepeated re roots. $Q(s) = (s-\lambda_1)\cdots(s-\lambda_n)$

$F(s) = \frac{k_1}{s-\lambda_1} + \cdots + \frac{k_n}{s-\lambda_n}$ $\quad k_i = (s-\lambda_i)F(s)\big|_{s=\lambda_i}$

E. $F(s) = \frac{2s^2+5}{s^2+3s+2} \overset{\cdots}{=} 2 + \frac{7}{s-1} + \frac{-13}{s-2}$

$f(t) = 2\delta(t) + (7e^{-t} - 13e^{-2t})u(t)$

E. $F(s) = \frac{\overset{1}{\overline{s+3}} + \overset{2}{\overline{3}}e^{-2s}}{(s+1)(s+2)} = f_1(t) + f_2(t-2)$ $\quad$ ($u(t-2)$ as well)

2. Unrepeated, cmplx conj. pair roots ✓ if real coeff.

$F(s) = \frac{As+B}{s^2+2as+b} = \frac{As+B}{(s-r)(s-r^*)} = \frac{k_1}{s-r} + \frac{k_2}{s-r^*} \overset{\cdots}{=} \frac{k}{s-r} + \frac{k^*}{s-r^*}$ $\quad$ (k_1, k_2 conj.)

$k = (s-r)F(s)\big|_{s=r} \to k^*$

$f(t) = (ke^{rt} + k^* e^{r^*t})u(t)$ $\quad$ Let $r \equiv \overset{\text{car}}{x + jy}$, $k \equiv \overset{\text{pol}}{\rho e^{j\theta}}$

$= (\rho e^{j\theta} e^{xt+jyt} + \rho e^{-j\theta} e^{xt-jyt})u(t)$

$= \rho e^{xt}(e^{j(yt+\theta)} + e^{-j(yt+\theta)})u(t)$

$= \rho e^{xt}\cdot 2\cos(yt+\theta)\, u(t)$

E. $F(s) = \frac{6(s+34)}{s(s^2+10s+34)} = \frac{k}{s} + \frac{c}{s+5+3j} + \frac{c^*}{s+5-3j}$

$\overset{\cdots}{=} \frac{6}{s} + \frac{-3-4j}{s+5+3j} + \frac{-3+4j}{s+5-3j}$ $\quad$ $\left(-3+4j = 5e^{j(\tan^{-1}\frac{4}{3} - \pi)}\right)$

$= 6u(t) + 2\cdot 5e^{-5t}\cos\left(-3t + \tan^{-1}\frac{4}{3} - \pi\right)u(t)$

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3. Repeated

$$F(s) = \frac{P(s)}{(s-\lambda)^r (s-\alpha_1)\cdots(s-\alpha_n)}$$

$$= \frac{a_0}{(s-\lambda)^r} + \frac{a_1}{(s-\lambda)^{r-1}} + \cdots \frac{a_{r-1}}{s-\lambda} + \frac{k_1}{s-\alpha_1} + \cdots$$

$$a_0 = (s-\lambda)^r F(s) \Big|_{s=\lambda}$$

$$a_1 = \frac{d}{ds}\left[(s-\lambda)^r F(s)\right]_{s=\lambda}$$

Pf. $(s-\lambda)^r F(s) = \cancel{a_0} + a_1(s-\lambda) + a_2(s-\lambda)^2 + \cdots$

$$a_m = \frac{1}{m!}\frac{d^m}{ds^m}\left[(s-\lambda)^r F(s)\right]_{s=\lambda}$$

$\frac{d}{ds}\left[(s-\lambda)^r F(s)\right] = a_1 + 2a_2 \underbrace{\cancel{(s-\lambda)}}_{0} + \cancel{\cdots}$

$$\mathcal{L}^{-1}\left[\frac{1}{(s-\lambda)^2}\right] = \mathcal{F}^{-1}\left[\frac{d}{ds}\left(-\frac{1}{s-\lambda}\right)\right] = t e^{\lambda t} u(t)$$

$$\mathcal{L}^{-1}\left[\frac{1}{(s-\lambda)^n}\right] \overset{\cdots}{=} = \frac{1}{(n-1)!} t^{n-1} e^{\lambda t} u(t)$$

System analysis

$$\begin{matrix} f(t)u(t) \to \\ \cancel{y\,\dot{y}(0^-)} \to \end{matrix} \boxed{LTI} \longrightarrow y(t) = y_{zs}(t) + y_{zi}(t)$$

E. $(D^2+5D+6)\,y(t) = (D+1)\,f(t)$

$y(0^-) = 2, \quad \dot{y}(0^-) = 1$

$f(t) = e^{-4t}u(t)$ (causal)

f causal

$$s^2 Y - s\,y(0^-) - \dot{y}(0^-) + 5(sY - y(0^-)) + 6Y = sF - \cancel{f(0^-)} + F$$

$$(s^2+5s+6)\,Y = (s+1)\,F + \left[(s+5)\,y(0^-) + \dot{y}(0^-)\right]$$

$$Y(s) = \frac{(s+1)\,F}{s^2+5s+6} + \frac{(s+5)\,y(0^-) + \dot{y}(0^-)}{s^2+5s+6}$$

$$\equiv Y_{zs}(s) + Y_{zi}(s)$$

$F(s) = \frac{1}{s+4}$

$$= \frac{s+1}{(s+2)(s+3)(s+4)} + \frac{2s+11}{(s+2)(s+3)}$$

$$Y = \frac{-1/2}{s+2} + \frac{2}{s+3} + \frac{-3/2}{s+4} + \frac{7}{s+2} - \frac{5}{s+3}$$

$$y(t) = \left(-\tfrac{1}{2}e^{-2t} + 2e^{-3t} - \tfrac{3}{2}e^{-4t}\right)u(t) + \left(7e^{-2t} - 5e^{-3t}\right)u(t)$$

$y_{zs}(t) = f(t) * \overset{?}{h(t)}$ Can find h from DFQ?

$$Q(D)\,y(t) = P(D)\,f(t)$$

$\mathcal{L}$ $\quad Q(s)\,Y_{zs}(s) = P(s)\,F(s)$ init $=0$

$H = \frac{Y_{zs}}{F} = \frac{P(s)}{Q(s)}$ } transfer function $H(s) \xrightarrow{\mathcal{L}^{-1}} h(t)$

E. (continued) $\quad H(s) = \frac{s+1}{s^2+5s+6} \overset{\cdots}{=} \frac{-1}{s+2} + \frac{2}{s+3}$

$$h(t) = \left(-e^{-2t} + 2e^{-3t}\right)u(t)$$

System stability

$y(t) = \underbrace{y_{zi}(t)}_{1.\text{ asy}} + \underbrace{y_{zs}(t)}_{2.\text{ BIBO}}$

1. Asymptotic

will $y_{zi}(t)$ die out? (input $f = 0$)

Def. A sys. is asy. stable if $y_{zi}(t) \to 0$ as $t \to \infty$ $\forall i$

unstable if $|y_{zi}(t)| \to \infty$ —— $\exists i$

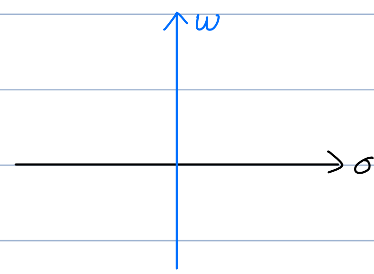
marginally stable if $y_{zi}(t)$ remains bounded $\forall i$

Conditions $H(s) = \frac{P(s)}{Q(s)}$ roots of $P(s) \xrightarrow{\text{def.}}$ zeroes

roots of $Q(s) \xrightarrow{\text{def.}}$ poles

$Y_{zi}(s) = \frac{\sim \text{init}}{Q(s)}$. Want $\exp \to 0$

Stable $\forall \overset{\text{pole}}{\lambda}$; $\mathrm{Re}[\lambda] < 0$



Unstable 1. $\exists \lambda$; $\mathrm{Re}[\lambda] > 0$

2. $\exists$ repeated λ; $\mathrm{Re}[\lambda] = 0$

m. stable $\forall \lambda; \mathrm{Re}[\lambda] \leq 0$, $\exists$ unrepeated λ; $\mathrm{Re}[\lambda] = 0$

E. $(D-1)(D^2+4D+8)\, y(t) = (D-3)\, f(t)$

$\lambda = +1$ $\lambda = -2 \pm 2j$ $\Longrightarrow$ unstable

E. $(D+2)(D^2+4)^2\, y(t) =$ ~~~

$\lambda = -2$ $\lambda = \underset{}{\overset{\times 2\ \times 2}{\pm 2j}}$ $\Longrightarrow$ unstable

2. BIBO stability

(bounded input bounded output)

BI $\xrightarrow{\text{always}}$ BO

Thm. a sys. is BIBO stable iff $\int_{-\infty}^{\infty} |h(t)|\, dt$ exists (abs $\int$able)

Pf. $\Leftarrow$ BI $\to |f(t)| \leq K$ $\forall t$, and $\int |h|$ exists

$$y_{zs}(t) = h * f$$

$$= \int_{-\infty}^{\infty} h(\tau) f(t-\tau)\, d\tau \leq \int_{-\infty}^{\infty} |h(\tau)| \cdot |f(t-\tau)|\, d\tau$$

$$\leq K \int_{-\infty}^{\infty} |h(\tau)|\, d\tau \to \text{BO}$$

$\cancel{\Leftarrow}$ if $\int_{-\infty}^{\infty} |h(t)|\, dt = \infty$ Let $f(t) = \mathrm{sgn}(h(-t))$ (BI)

$$y(0) = \int_{-\infty}^{\infty} h(\tau) f(0-\tau)\, d\tau$$

$$= \int_{-\infty}^{\infty} h(\tau)\, \mathrm{sgn}(h(\tau))\, d\tau$$

$$= \int_{-\infty}^{\infty} |h(\tau)|\, d\tau \equiv \infty$$

BI $\to$ ~~BO~~ (~~BO~~)

$1 \overset{?}{\leftrightarrow} 2$ Asy $\Rightarrow$ BIBO (all exp)

Asy. m. $\Rightarrow$ ~~BIBO~~ ($\int |\sin(t)| dt \to \infty$)

BIBO $\not\Rightarrow$ Asy. (asy. stronger) (zero/pole cancel). both cancel,

LTI sys. implementation (block diagram)

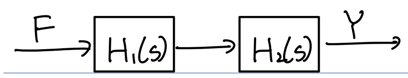
blocks $\overline{\ a\ }$ multiplier to realize $H(s)$

Σ accumulator

$\boxed{\frac{1}{s}}$ integrator

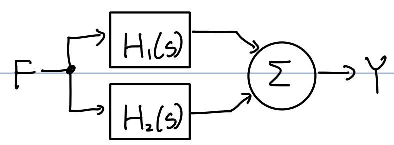
Ele. interconnections $F(s) \to \boxed{H(s)} \to Y(s)$

1. Cascade



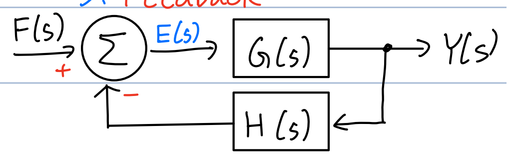
$Y = (H_1 H_2) F$

2. Parallel



$Y = (H_1 + H_2) F$

3. Feedback



$E = F - HY$

$Y = GE = GF - GHY$

$H_{eff} = \frac{Y}{F} = \frac{G}{1+HG}$

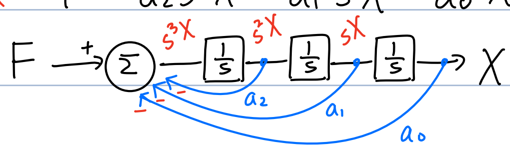
1. Direct-form realization

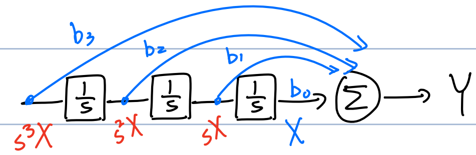
E. $H(s) = \frac{b_3 s^3 + b_2 s^2 + b_1 s + b_0}{s^3 + a_2 s^2 + a_1 s + a_0}$ cascade $\frac{\text{num}}{\text{den}}$ $F \to \boxed{H_1} \xrightarrow{X} \boxed{H_2} \to Y$

$= \Big(\underbrace{\frac{1}{s^3 + a_2 s^2 + a_1 s + a_0}}_{H_1(s)}\Big)\underbrace{(b_3 s^3 + b_2 s^2 + b_1 s + b_0)}_{H_2(s)}$

$F = s^3 X + a_2 s^2 X + a_1 s X + a_0 X$ $X(b_3 s^3 + b_2 s^2 + b_1 s + b_0) = Y$

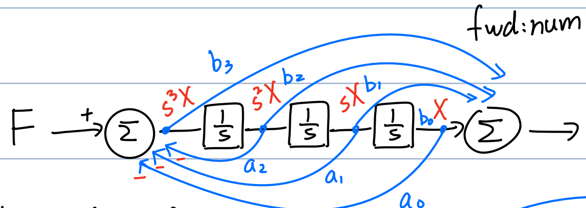
$s^3 X = F - a_2 s^2 X - a_1 s X - a_0 X$



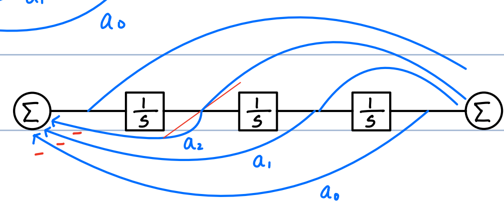


Canonical, can share $\boxed{\frac{1}{s}}$

fwd: num

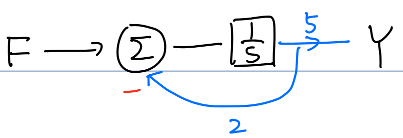


back: den. (all −)



Always make sure $a_3 = 1$!

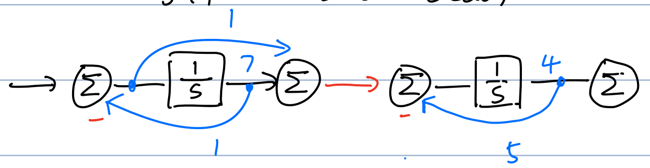
E $H(s) = \frac{5}{s+2}$

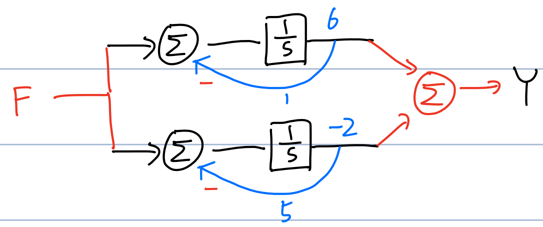


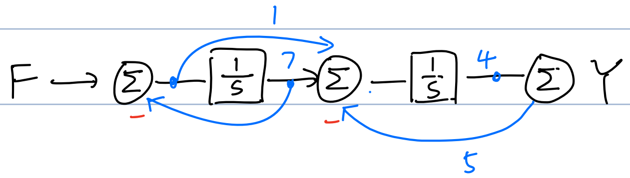
2. Cascade / parallel form

E. $H(s) = \dfrac{4s+28}{(s+1)(s+5)}$ → factor!

$= \dfrac{s+7}{s+1} \cdot \dfrac{4}{s+5}$ (cas) $\quad \therefore = \dfrac{6}{s+1} + \dfrac{-2}{s+5}$ (∥)

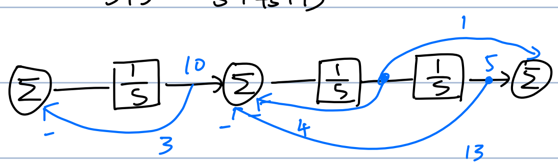


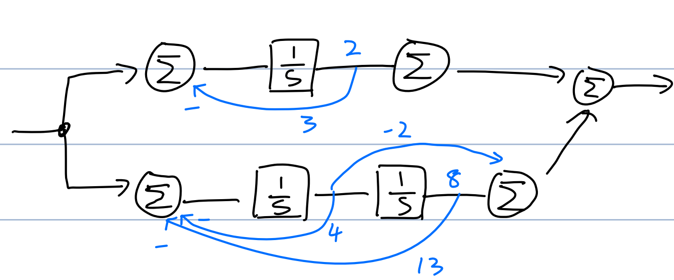




E. $H = \dfrac{10s+50}{(s+3)(s^2+4s+13)}$ all # real!

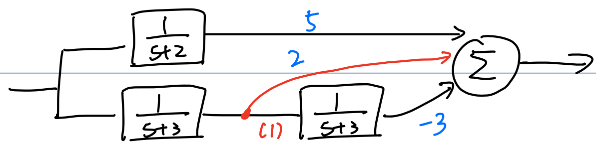
$= \dfrac{10}{s+3} \cdot \dfrac{s+5}{s^2+4s+13}$ $\quad \therefore = \dfrac{2}{s+3} + \dfrac{\overset{As+B}{-2s+8}}{s^2+4s+13}$

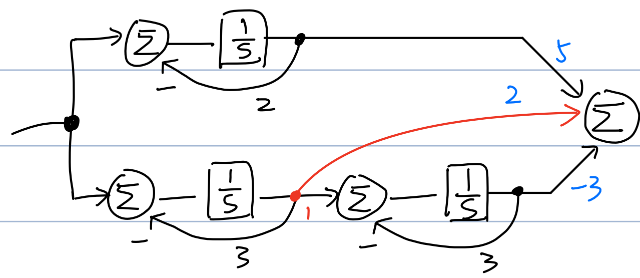




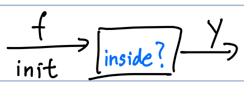
E. $H(s) = \dfrac{7s^2+37s+51}{(s+2)(s+3)^2}$

$\therefore = \dfrac{5}{s+2} + \dfrac{2}{s+3} + \dfrac{-3}{(s+3)^2}$ → same pole!





State-space rep.



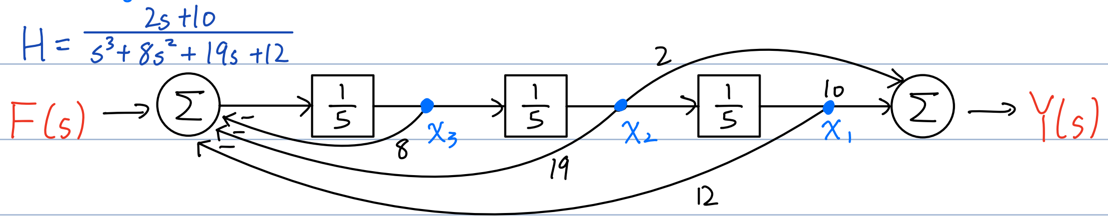
Def. The state of a sys. at any time t_0 is the smallest set of #s $\{x_1(t_0), \dots, x_n(t_0)\}$ that is sufficient to determine sys. behavior $\forall t > t_0$ when input $f(t)$ given, for $t > t_0$

if $t_0 = 0$, init.

$x_1, \dots, x_n$ are state variables, always integrator output!

Trans. fucn. → state eq. (E.)

1. Canonical: $H = \dfrac{2s+10}{s^3+8s^2+19s+12}$



① State eq.

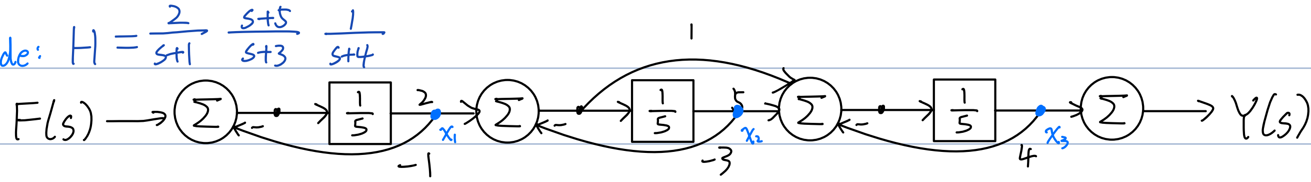
$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = x_3$$
$$\dot{x}_3 = -12x_1 - 19x_2 - 8x_3 + f$$

$$\Rightarrow \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -12 & -19 & -8 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} f$$

$$\underline{\dot{x}} = A \quad \underline{x} + \underline{b} \quad f$$

② Output eq. $y = 10x_1 + 2x_2 \Rightarrow y = [10, 2, 0] \cdot \underline{x}$

2. Cascade: $H = \dfrac{2}{s+1} \; \dfrac{s+5}{s+3} \; \dfrac{1}{s+4}$



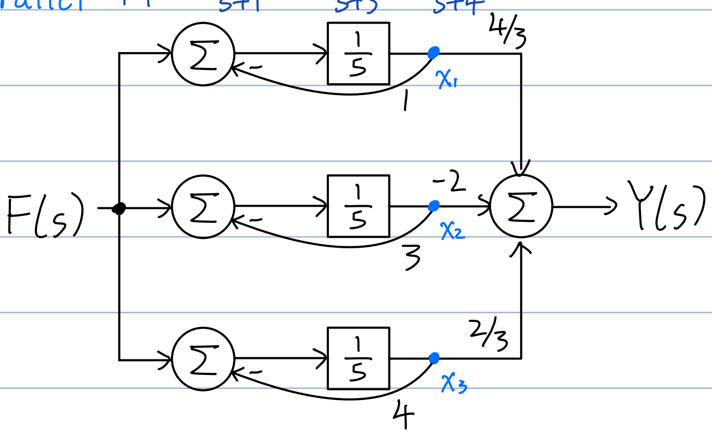
$$\dot{x}_1 = -x_1 + f$$
$$\dot{x}_2 = 2x_1 - 3x_2$$
$$\dot{x}_3 = 5x_2 - 4x_3 + \dot{x}_2 = 5x_2 - 4x_3 + 2x_1 - 3x_2$$
$$y = x_3$$

$$\Rightarrow \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 2 & -3 & 0 \\ 2 & 2 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} f$$

$$\Rightarrow y = [0, 0, 1] \cdot \underline{x}$$

cascade mats. are lower-triangular! ; $y = x_n$

3. Parallel: $H = \dfrac{4/3}{s+1} + \dfrac{-2}{s+3} + \dfrac{2/3}{s+4}$



$$\dot{x}_1 = -x_1 + f$$
$$\dot{x}_2 = -3x_2 + f$$
$$\dot{x}_3 = -4x_3 + f$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} f$$

$$y = \left[\tfrac{4}{3}, -2, \tfrac{2}{3}\right] \cdot \underline{x}$$

State vars. are decoupled

n^{th} order ⇒ 1^{st} order mat.

parallel: diagonal, entries are poles

(also see internals)

Freq. response $\quad H(\omega) = H(s)\big|_{s=j\omega}$ $\qquad$ $h(t)$ real, causal ?

E. ideal delay $\quad h(t) = \delta(t-T)$

$$H(s) = e^{-sT}$$

$$H(\omega) = e^{-j\omega T} \rightarrow |H(\omega)| = 1, \quad \measuredangle H(\omega) = -\omega T$$

E. ideal differentiator $\quad H(s) = s$

(symmetry) $\quad H(\omega) = j\omega \rightarrow |H(\omega)| = |\omega|, \quad \measuredangle H(\omega) = \frac{\pi}{2} \ (\omega>0); \ -\frac{\pi}{2} \ (\omega<0)$

E. ideal $\int$ $\quad H(s) = \frac{1}{s}$

$$H(\omega) = \frac{1}{j\omega} \rightarrow |H(\omega)| = \frac{1}{|\omega|}, \quad \measuredangle H(\omega) = -\frac{\pi}{2} \ (\omega>0); \ \frac{\pi}{2} \ (\omega<0)$$

Causal sinusoid response $\quad f(t) = e^{j\omega_0 t} u(t) \longrightarrow H(s) = \frac{P(s)}{Q(s)} \longrightarrow ?$

$$F(s) = \frac{1}{s-j\omega_0} \qquad Y_{zs}(s) = F(s)\,H(s)$$

additional pole $j\omega_0$ due to input

$$= \frac{1}{s-j\omega_0} \, \frac{P(s)}{(s-\lambda_1)\cdots(s-\lambda_n)}$$

$$\alpha = \overset{\mathcal{L}}{H(s)}\Big|_{s=j\omega_0} = \overset{\mathcal{F}}{H(\omega_0)} \qquad = \frac{\alpha}{s-j\omega_0} + \frac{k_1}{s-\lambda_1} + \cdots + \frac{k_n}{s-\lambda_n}$$

$$y_{zs}(t) = \underbrace{H(\omega_0)\,e^{j\omega_0 t} u(t)}_{\text{steady-state}} + \underbrace{\sum_n k_i e^{\lambda_i t} u(t)}_{\text{transient}}$$

$y_{ss}(t)$, scaled input $\quad$ $y_{tr}(t)$, decays for stable sys.

Similarly, if $f = \cos(\omega_0 t + \theta)\,u(t)$

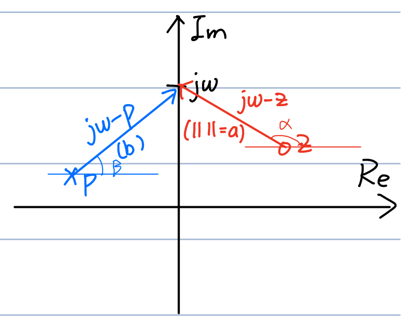
$$y_{zs} = |H(\omega_0)| \cos(\omega_0 t + \theta + \measuredangle H(\omega_0))\, u(t)$$

causal periodic $f \longrightarrow$ causal periodic y_{ss} + dying y_{tr}

Filter design $\quad H(s) = \frac{s-z}{s-p}$ $\quad$ ○ zero, × pole $\Rightarrow$ } freq. response

$$H(\omega) = \frac{j\omega - z}{j\omega - p}$$

$$|H(\omega)| = \frac{|j\omega - z|}{|j\omega - p|} = \frac{a}{b} \qquad \measuredangle H(\omega) = \alpha - \beta$$



E. $z=-1$, $p=-2$ (both real)

$$|H(\omega)| = \frac{\sqrt{1+\omega^2}}{\sqrt{4+\omega^2}} \qquad \left(H(0) = \tfrac{1}{2} \rightsquigarrow H(\infty) = 1\right)$$

$$\measuredangle H(\omega) = \ \sim \qquad (H(0) = 0 \rightsquigarrow H > 0 \rightsquigarrow H(\infty) = 0)$$

Sys. rep. 1. DFQ $Q(D)\, y(t) = P(D) f(t)$

2. impulse response $h(t)$

3. transfer $H(s) = \frac{P(s)}{Q(s)}$

4. block diagram (3)

5. state-space

Sys. anal. $H(s)$ + init $\Rrightarrow$ ~

$Y_{zs}(s) = H(s)\, F(s)$

$Y_{zi}(s) = \frac{\text{init. cond.}}{Q(s)}$

$y(t) = \mathcal{L}^{-1}[Y_{zs}(s) + Y_{zi}(s)]$