1. Integrator with Reset

Assume the switch is short when closed and open when opened.

(a) The circuit becomes an RC integrator.

$$i_c(t) = -C_1 \frac{dV_a(t)}{dt}$$

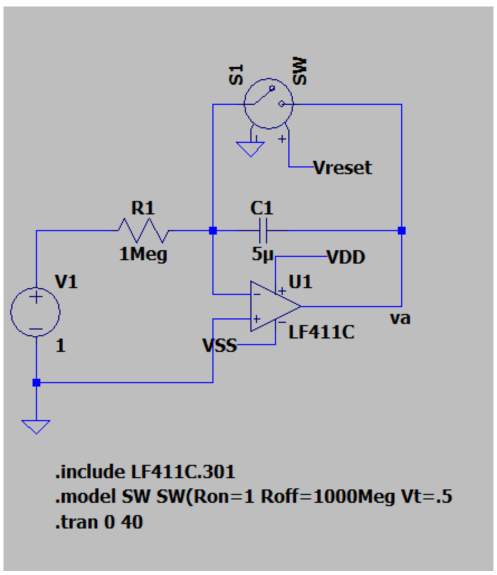
$$\frac{V_1(t)}{R_1} = -C_1 \frac{dV_a(t)}{dt}$$

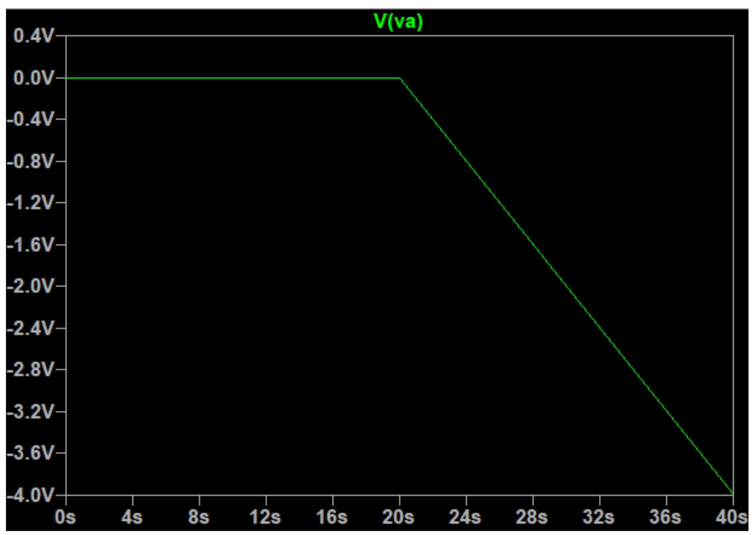
$$V_a(t) = -\frac{1}{R_1 C_1} \int_{-\infty}^{t} V_1(t')\,dt'$$

$$= 0 - \frac{1}{R_1 C_1} \int_{0}^{t} V_1(t')\,dt'$$

(b) When the switch is closed, va(t) = vN = vP = 0 V, since va(t) is shorted to vN, which is virtually shorted to vP, which is grounded.

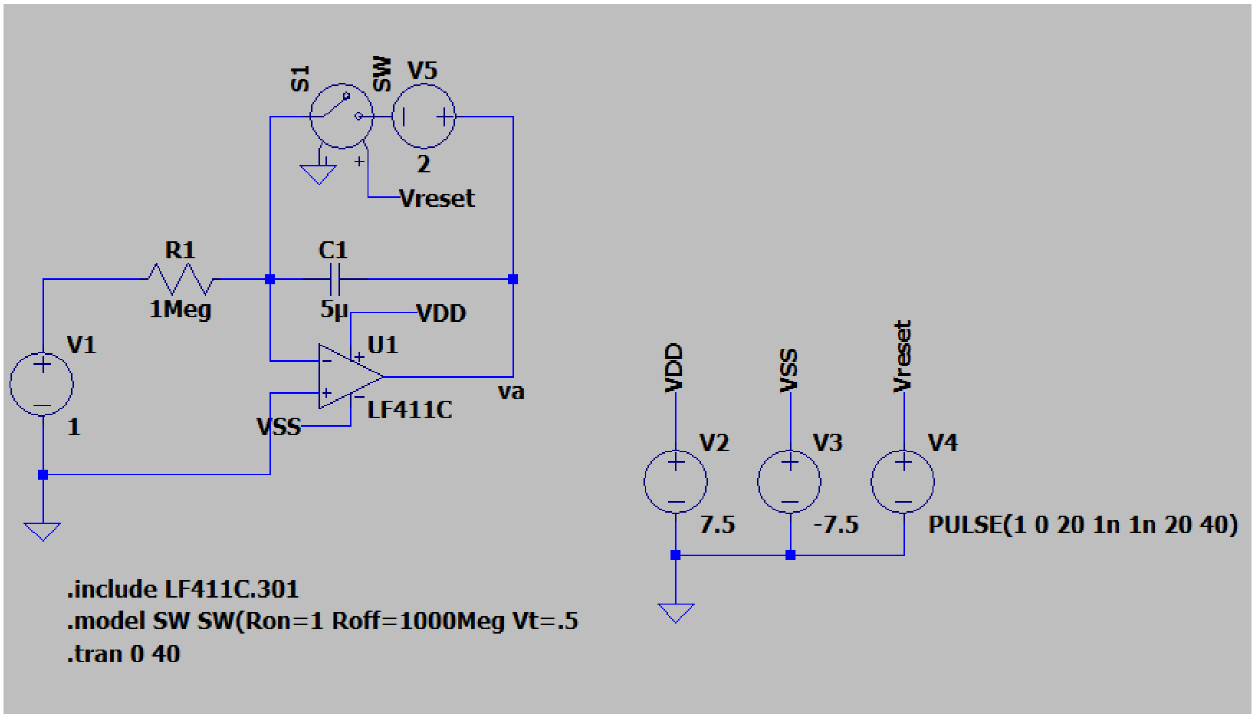
(c) The switch is closed at the beginning and opens at t = 20 s.
V1(t) is set to be 1 V DC.

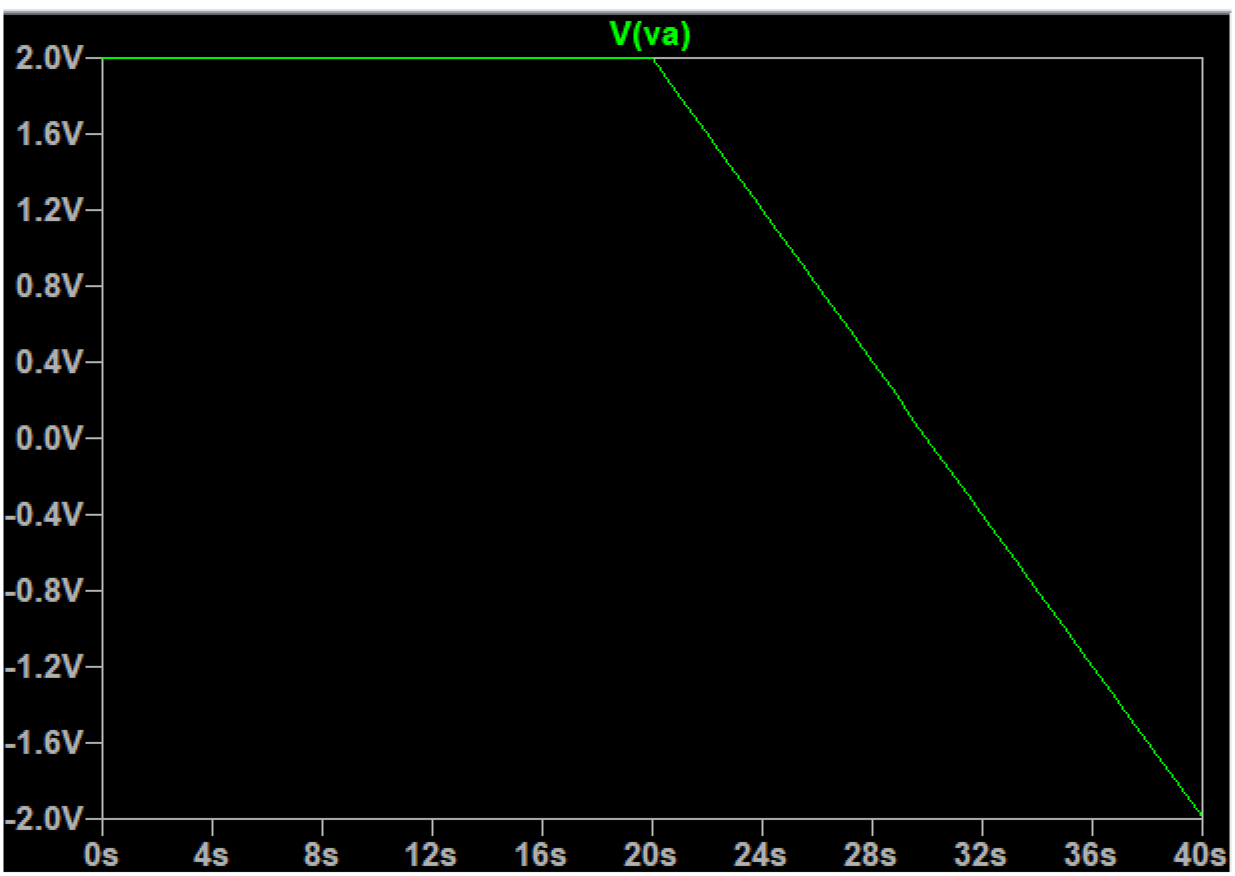




va(t) stays at 0 V when S is closed. After S opens, va(t) starts to integrate V1(t), with a gain of -1/(R1C1).

(d) We added a 2 V DC voltage source in series with the switch. When the switch is closed, va(t) is charged to 2 V.



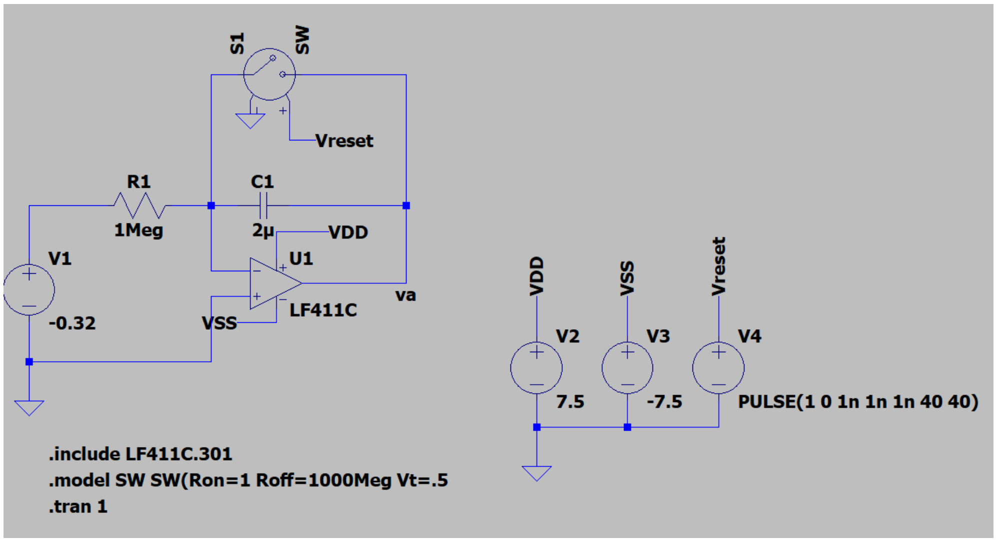


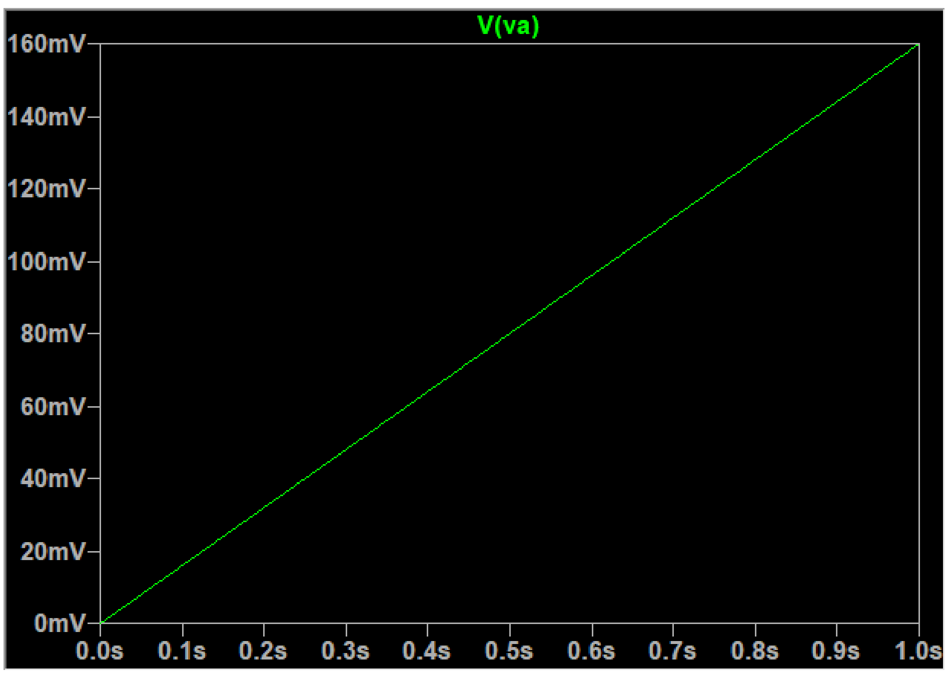
The curve shape is the same, except now va(0) = 2 V.

2. Velocity Integration

(a) We chose 1 V to map 100 ft/s^2 for acceleration, and -200 ft/s for velocity. $v(t) = v(0) + \int a(t)\, dt$. Since we need a velocity gain of -0.5, we need R1C1 = 2. R1 = 1 Meg and C1 = 2 µ suffices.

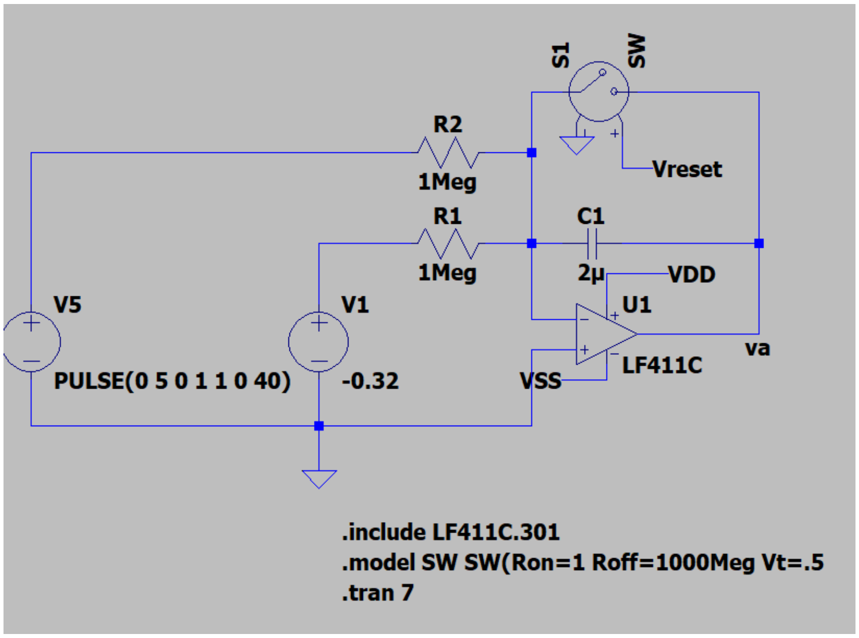
v1(t) = -0.32 V for gravity.

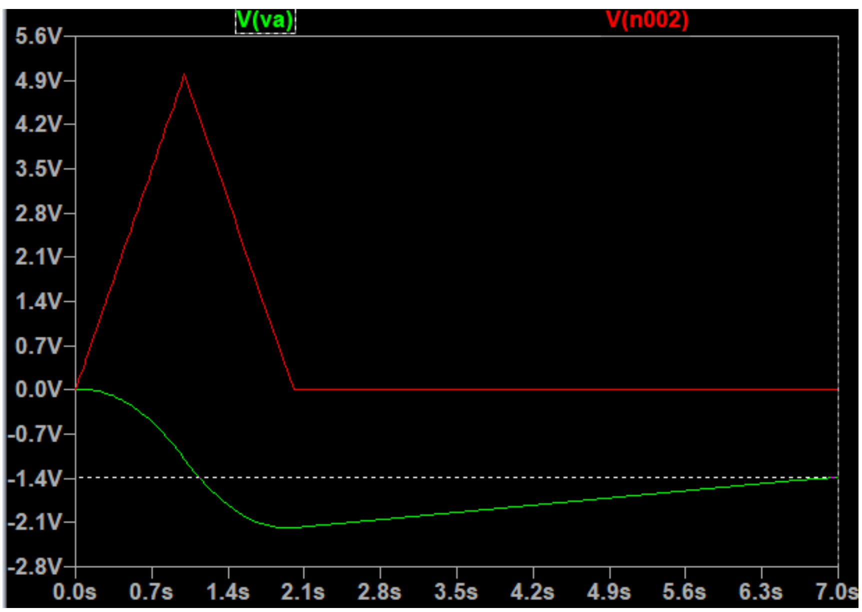




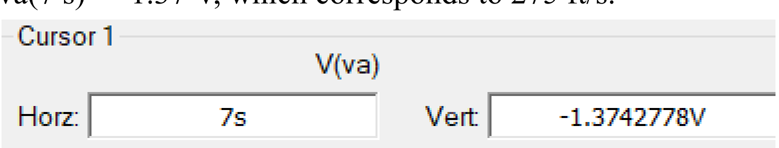
va(1 s) = 0.16 V, which corresponds to -32 ft/s.

(a) We used a PULSE (V5) source to model the acceleration, shown in red. We then connected the source, in series with another 1 Meg resistor, to the inverting input. By KCL, after the switch opens, the current from V1 and V5 will both flow through C1. va(t) = -0.5 ∫ (V1(t) + V5(t)) dt.





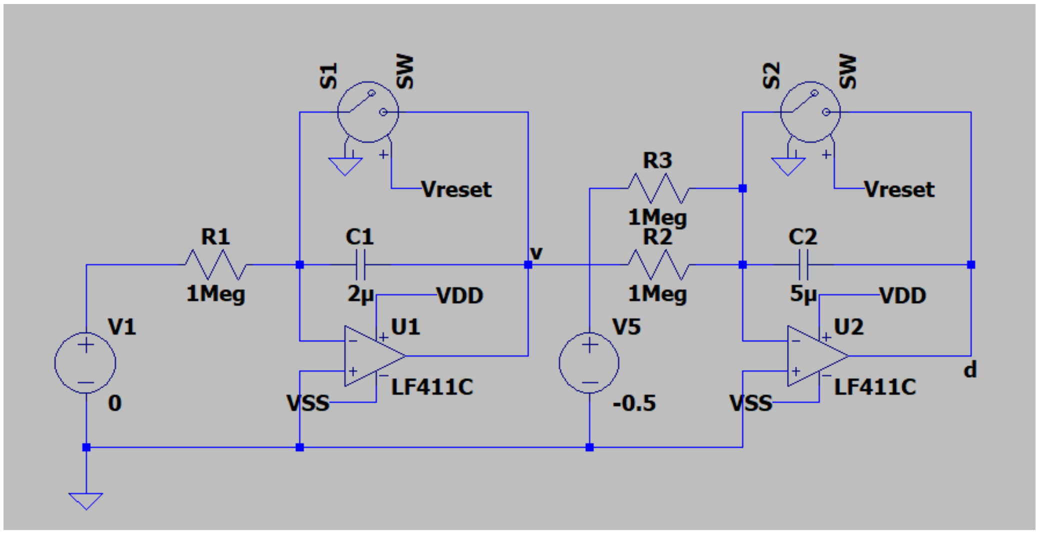
va(7 s) = -1.37 V, which corresponds to 275 ft/s.

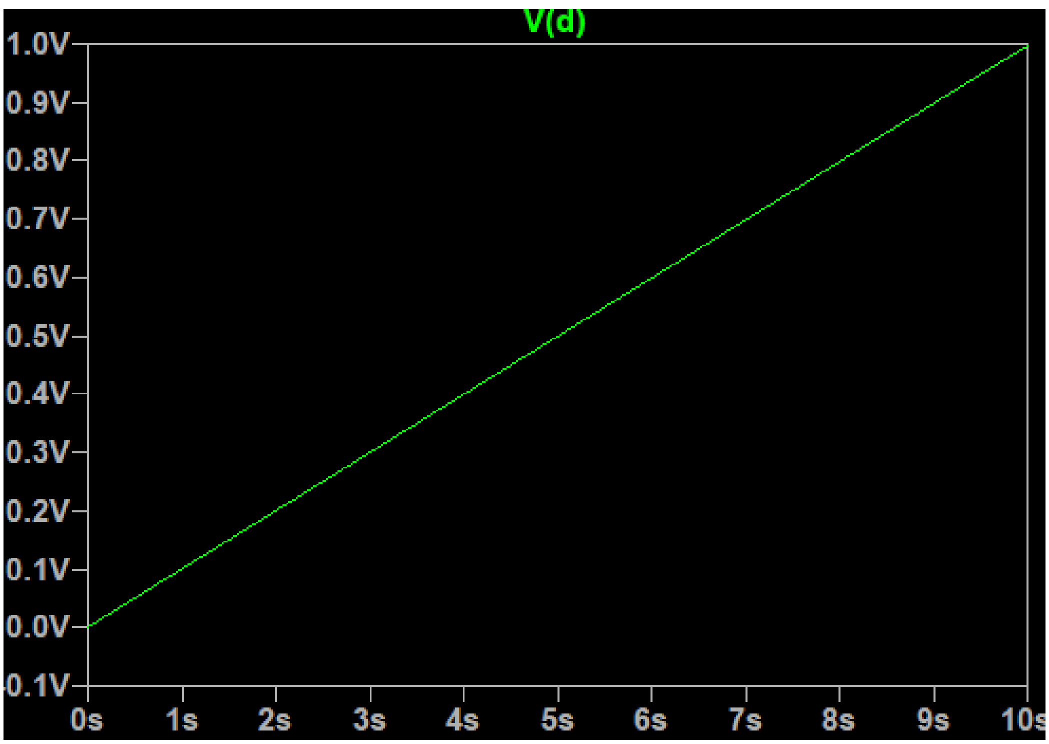


3. Distance Integration

(a) We chose 1 V to represent 1000 ft of distance.
$d(t) = d(0) + \int v(t)\, dt$. From the integral of velocity to distance, we need a gain of -0.2.
R2C2 = 5. R2 = 1 Meg and C2 = 5 μ suffices.

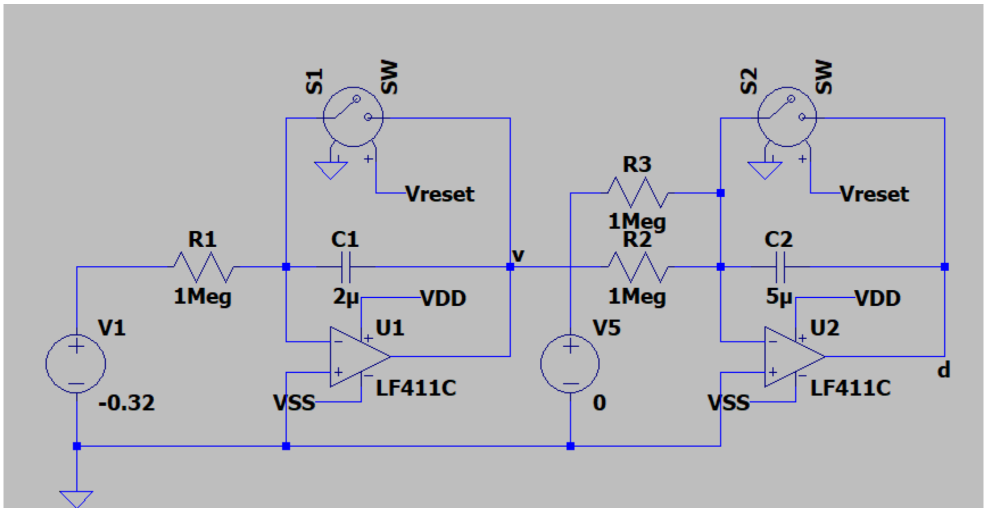
We set V1(t) = 0, so acceleration = 0, and the first stage (left portion) is voided.
V5(t) = -0.5 V to represent an initial velocity of 100 ft/s.

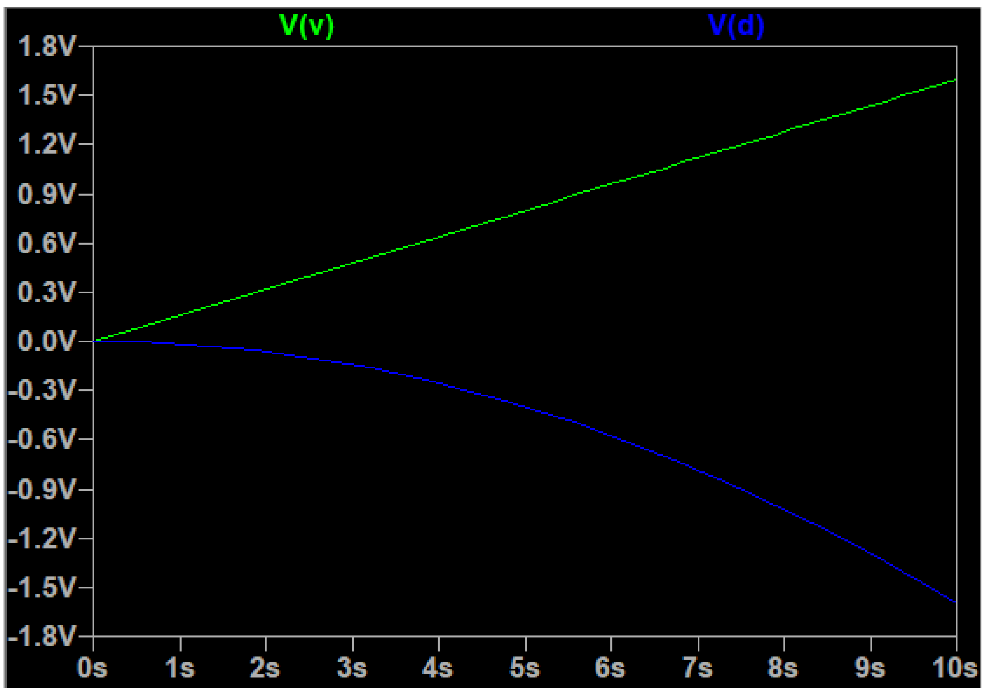




The final output voltage is 1 V, which represents 1000 ft.

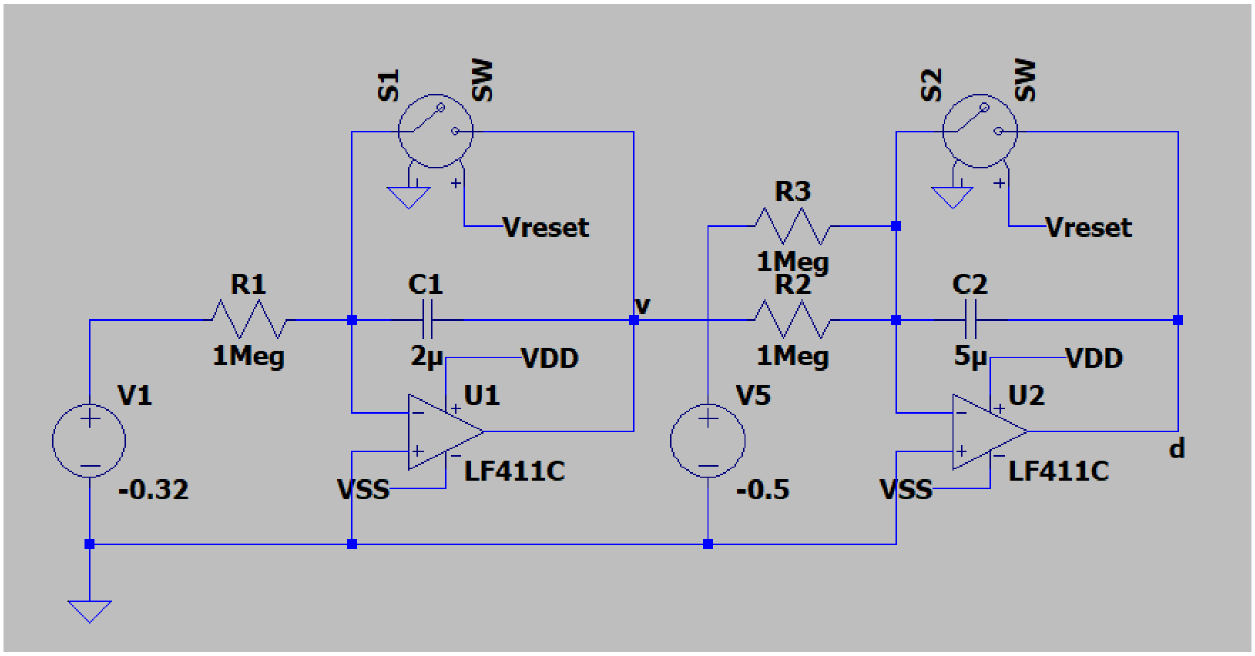
(b) Now we set V1(t) = -0.32 V for gravity, and V5(t) = 0 (no initial velocity, passive fall).

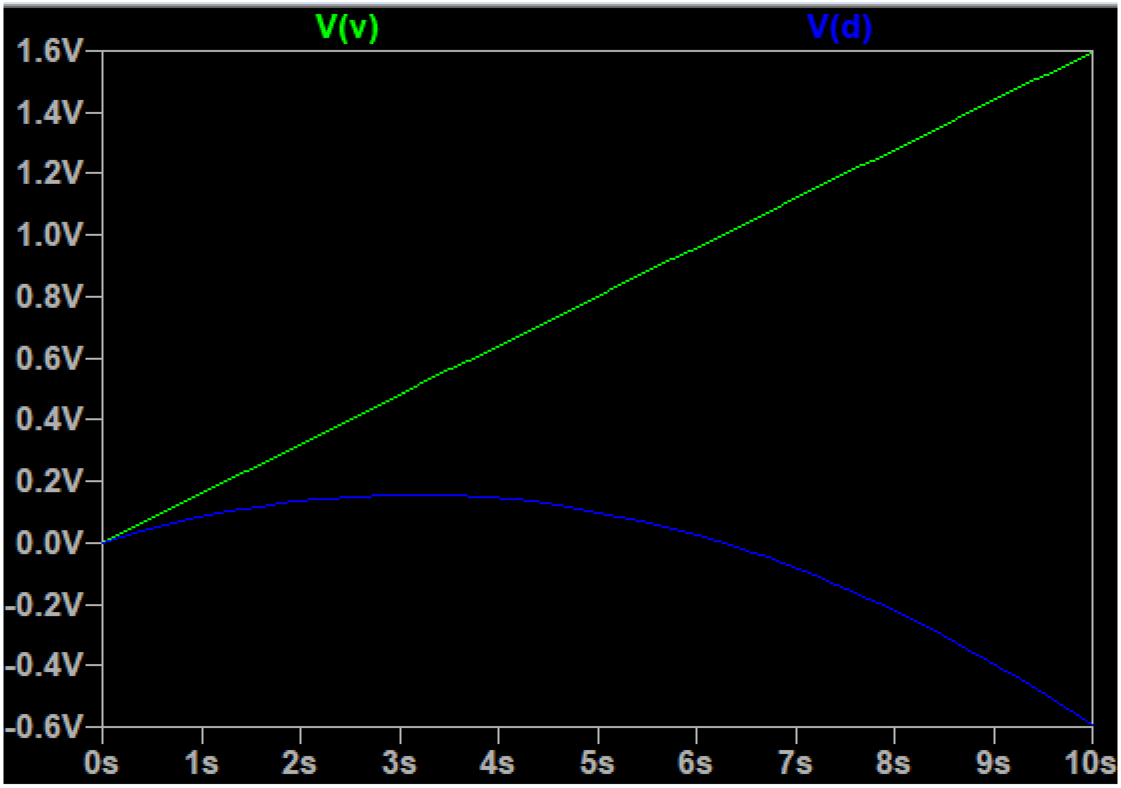




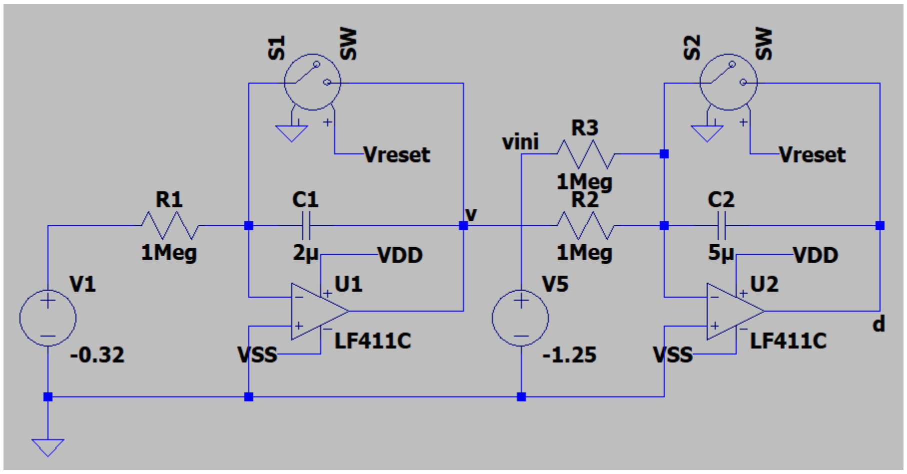
v(10 s) = 1.6 V, which maps to -320 ft/s.
d(10 s) = -1.6 V, which maps to -1600 ft.

(c) We added a power supply to the second integrator stage, as in part 2(b). Here the initial velocity is set to be 100 ft/s, same as part 3(a).

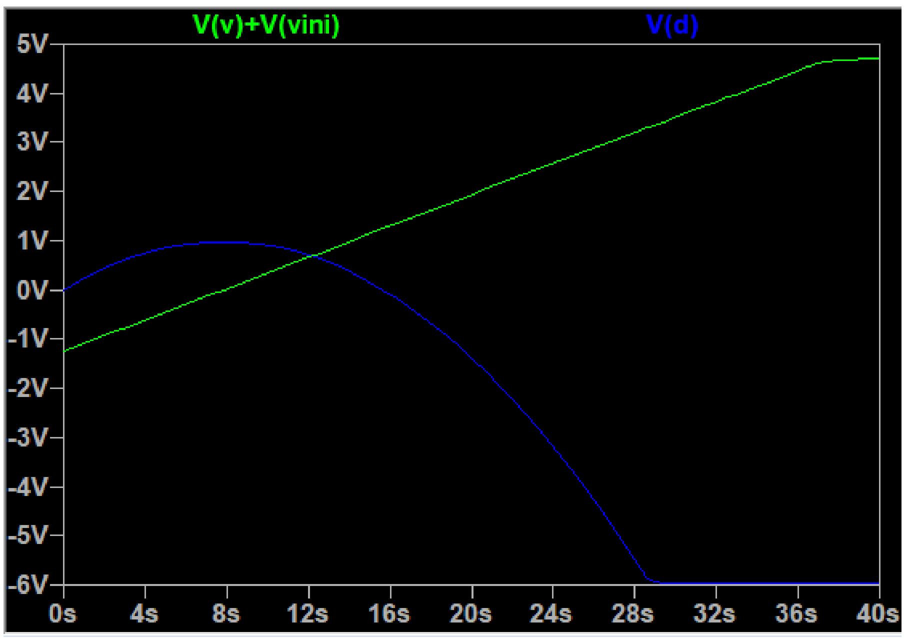




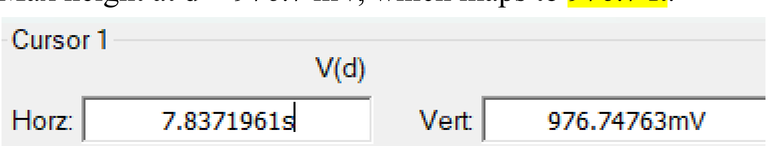
(d) V5(t) is set to be -1.25 V, which maps to +250 ft/s.



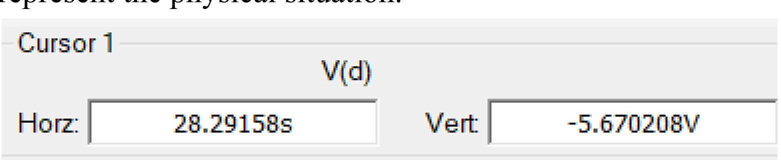
The velocity would be v(t) + vini(t), where v(t) is due to gravitational acceleration, and vinit(t) is due to initial velocity.



Max height at d = 976.7 mV, which maps to 976.7 ft.

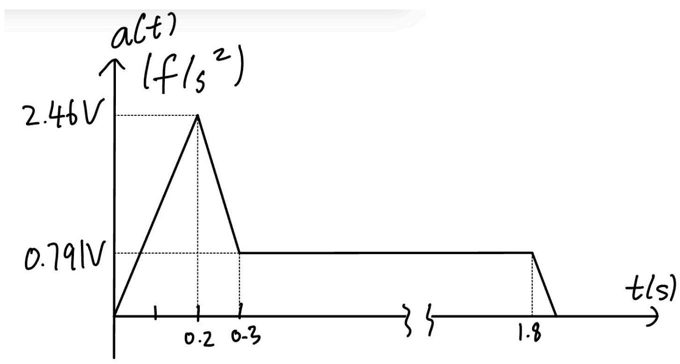


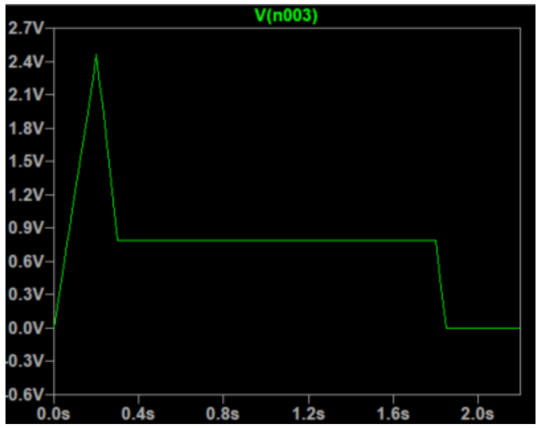
At around 28.3 s, the second op amp (distance) saturates. The output flattens out. It cases to represent the physical situation.



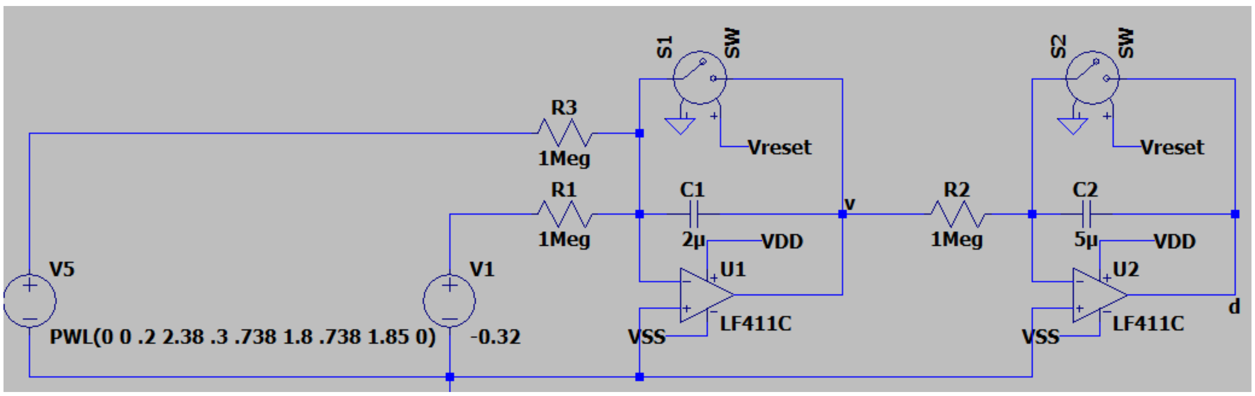
4. Rocket Flight Model

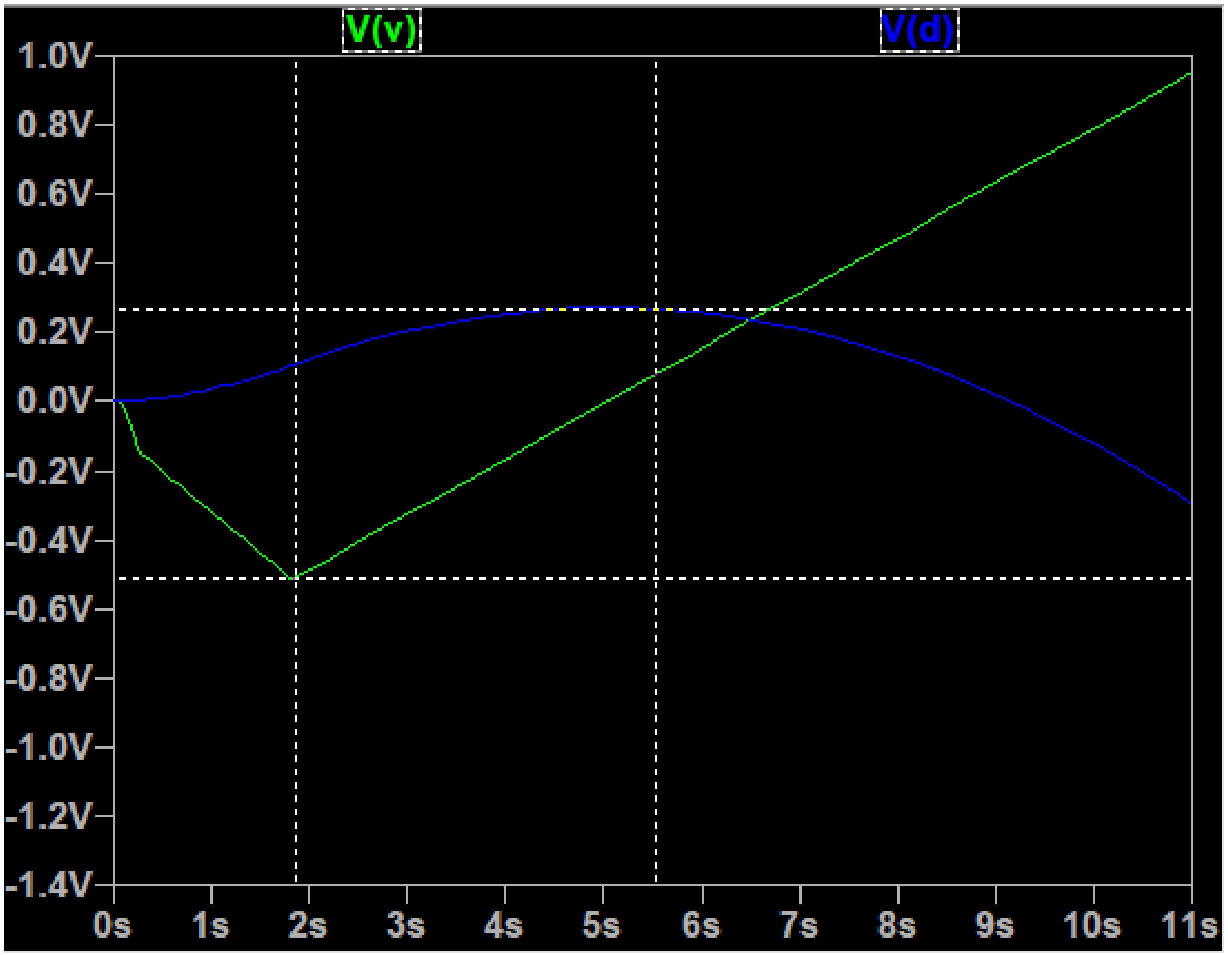
We first need to model the rocket thrust curve. a(ft/s^2) = F/0.2 kg * 3.28084 m/ft
Below is the acceleration vs. time model for our C6 engine.

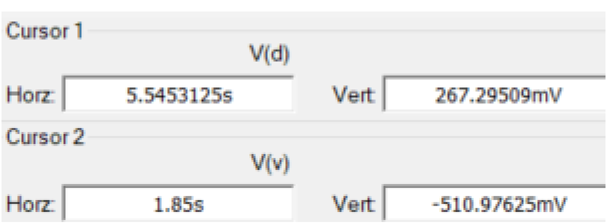




On LTSpice, we modeled the acceleration as a piecewise linear function:
PWL(0 0 .2 2.46 .3 .791 1.8 .791 1.85 0)





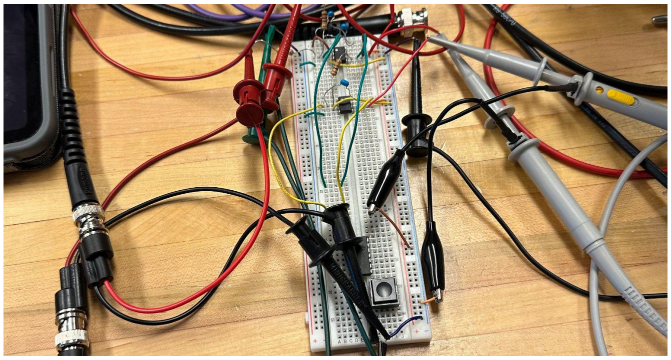


Maximum height: 267.30 mV, which maps to 267.30 ft.
Maximum velocity: -510.98 mV, which maps to 102.196 ft/s
Duration: 9.24 s

5. Physical Experiment

In the physical experiment, we used 4.7 uF capacitors instead of 5 uF in the second stage. The output voltage for distance will be slightly higher, but the difference is not significant.
The yellow curve is always the thrust acceleration, the green curve represents the velocity, and the blue curve represents height.

Here's a picture of our final two-stage integrator circuit:



1. Integrator

When the switch is closed, the output voltage stays at 0 V.

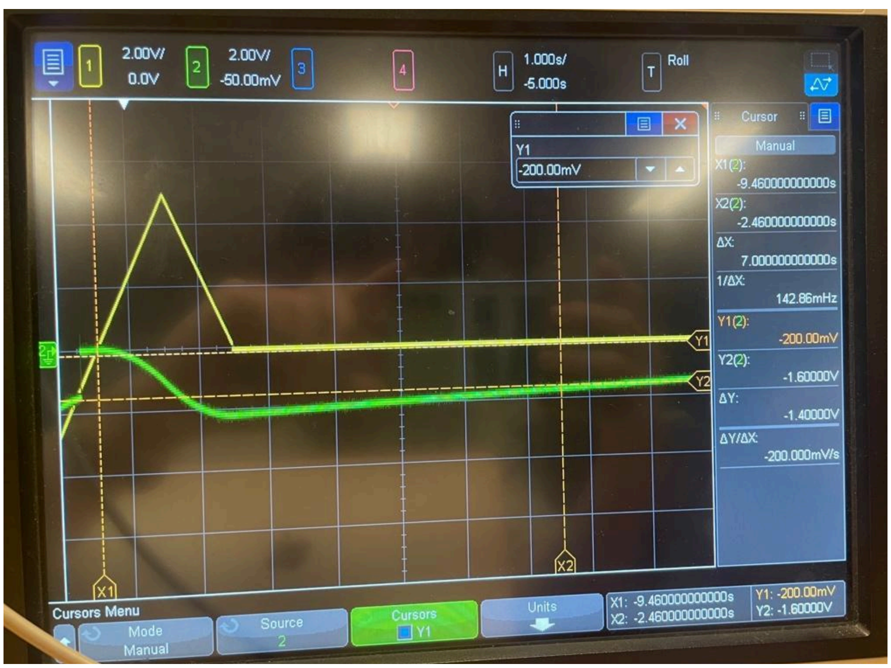
As the switch opens, the output voltage begins to integrate the input voltage.

2. Velocity Integration

(a) Same circuit setup as in part 2(a)

The switch opens at t = 0 s. The output voltage at t = 1 s is 0.168 V, which maps to -33.6 ft/s.

(b) Same circuit setup as in part 2(b), except with another triangular pulse source connected to the input of the op amp.



The output voltage at t = 7 s is -1.4 V, which maps to 280 ft/s.

3. Distance Integration

(a) Same setup as in part 3(a), with the second stage only

v(10 s) = 0.9375 V, which maps to 937.5 ft.

(b) Also same setup

v(10 s) = 1.66875 V, which maps to -333.75 ft/s.
d(10 s) = -1.68750 V, which maps to -1687.50 ft.

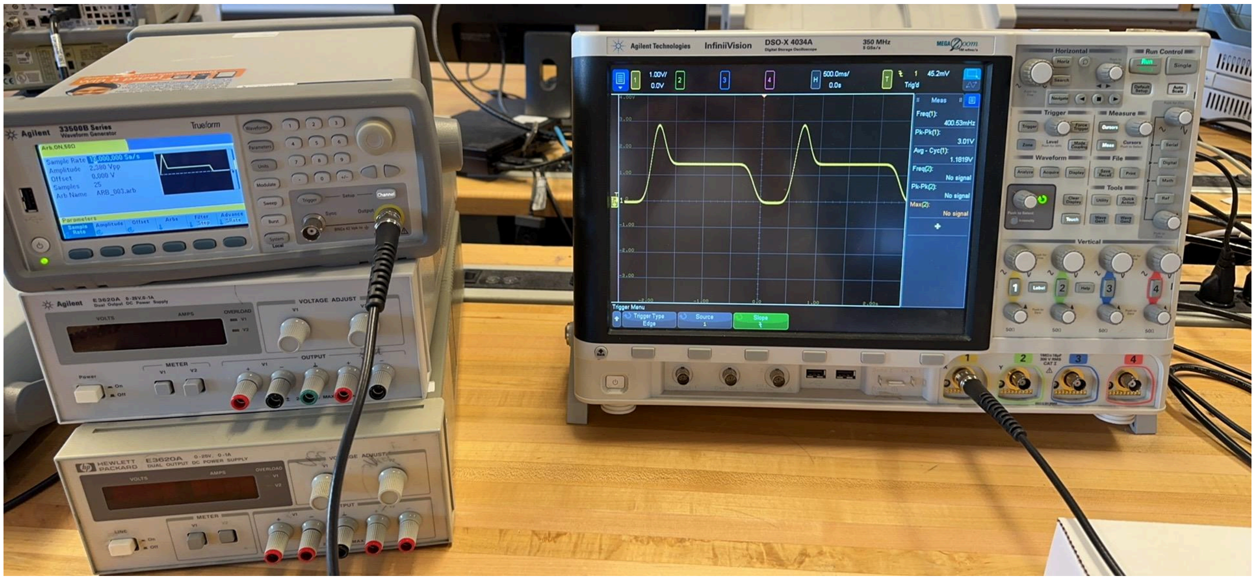
(d)

The maximum height is 0.975 V, which maps to 975 ft.

At around 29.3 s, the output voltage representing distance saturates and ceases to work.

4. Rocket Flight Model

(a) We created an arb pulse from the function generator. Due to undersampling, our waveform is not exactly triangular, but it is closely models the rocket thrust.



Max Height = 287 mV, which maps to 287 ft.

Max Velocity = -562.5 mV, which maps to 112.5 ft/s

Duration of flight = 9.46 s