$\cos a \cos b = \frac{1}{2}(\cos(a+b) + \cos(a-b))$

$\cos(a+b) = \cos a \cos b - \sin a \sin b$

AC time → phasor (sinusoidal domain) L's $\underline{I}$ lags $\underline{V}$

Gen. sin. $v(t) = V_m \cos(wt+\phi) = V_m \cos\phi \cos wt - V_m \sin\phi \sin wt$

$v'(t) = w V_m \cos(wt + \phi + 90^\circ)$ $\rightarrow jw\underline{X}$

$\int v(t)\,dt = \frac{1}{w} V_m \cos(wt+\phi - 90^\circ)$ $\rightarrow \frac{1}{jw}\underline{X}$

E. sin excitation → $y(t) = y_F(t) + y_N(t)$

$y_F(t) = A\cos wt + B \sin wt$ $y_N(t)$, turn source off, w/ i.v.

Stable lin. circuit → sin steady state, same w

Cmplx $j\, r\angle\theta = r\underline{\angle(\theta+90^\circ)}$

Phasor $v(t) = V_m \cos(wt+\phi) \longleftrightarrow \underline{V} = V_m \angle\phi$ w lost !

$= |\underline{V}| \cos(wt + \measuredangle \underline{V})$

Rotating phasor $\underline{V} = V_m \angle\phi$

$\underline{V} e^{jwt} = V_m e^{j(\phi + wt)}$

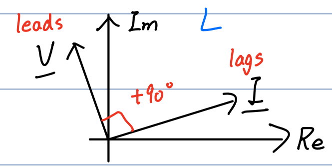
$= \underbrace{V_m \cos(wt+\phi)}_{\downarrow} + j V_m \sin(wt+\phi)$

$v(t) = \mathrm{Re}(\underline{V} e^{jwt})$

If $\mathrm{Re}(\underline{V_A} e^{jwt}) = \mathrm{Re}(\underline{V_B} e^{jwt})$ $\forall t$, $\longleftrightarrow$ $\underline{V_A} = \underline{V_B}$

sum if $\sum_{i=1}^{N} V_{m_i} \cos(wt + \phi_n) = V_m \cos(wt+\phi)$

$\sum_{i=1}^{N} \underline{V_i} = \underline{V}$



Impedance $\underline{I} = I_m \measuredangle \phi_I$, $\underline{V} = V_m \measuredangle \phi_V$ $\underline{Z} \equiv \frac{\underline{V}}{\underline{I}}$

		$(\underline{Z})$	$(\underline{Y})$	
R	$v(t) = R\,i(t)$	$\rightarrow \underline{V} = R\,\underline{I}$	$\underline{I} = G\,\underline{V}$	(same $\measuredangle$)
L	$v(t) = L\frac{di(t)}{dt}$	$\rightarrow \underline{V} = jwL\,\underline{I}$	$\underline{I} = \frac{1}{jwL}\underline{V}$	+90° → left shift → lead (v leads by 90, i lags)
C	$v(t) = \frac{1}{C}\int i(t)\,dt$	$\rightarrow \underline{V} = \frac{1}{jwC}\underline{I}$	$\underline{I} = jwC\,\underline{V}$	(v lags by 90)

$\underline{Z} \equiv R + jX$ R: resistance ; X: reactance $= \mathrm{Im}(\underline{Z})$

Admittance $\underline{Y} \equiv \frac{1}{\underline{Z}} = \frac{\underline{I}}{\underline{V}} = G + jB$ G: conductance; B: succeptance

$G = \frac{R}{R^2+X^2} \neq \frac{1}{R}$ $B = -\frac{X}{R^2+X^2} \neq -\frac{1}{X}$

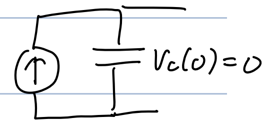
KVL $\sum_n v_n(t) = 0 \rightarrow \sum_n \underline{V_n} = 0$

KCL $\sum_n \underline{I_n} = 0$

Dep. source same thing, same unit (real consts.)

Source trans. $\underline{Z}_p = \underline{Z}_s$, $\underline{I}_s = \frac{\underline{V}_s}{\underline{Z}_s}$

Pathological E. no sin steady state, $V_c = 0$ forces a DC, never dies out! (w/o R)



E. RF wire inductance → balance w/ $\frac{1}{wC} = wL$ in series (resonance)

Node-V $\underline{I} = \frac{\underline{V_x} - \underline{V_y}}{\underline{Z}}$

Mesh-I $\underline{V} = \underline{Z}(\underline{I_x} - \underline{I_y})$

Superposition 1. All indep. same w ✓

2. diff. w ?? → $\underline{Z}$ differ from sources → find (t) → add

Thevenin $\underline{V_{oc}}$ series w/ $\underline{Z_{Th}}$

Norton $\underline{I_{sc}}$ parallel

$\underline{I_{sc}}\,\underline{Z_{Th}} = \underline{V_{oc}}$

Active same thing, just with $\underline{Z_1}$ E.i.: $\frac{\underline{V_o}}{\underline{V_I}} = -\frac{\underline{Z_F}}{\underline{Z_I}}$

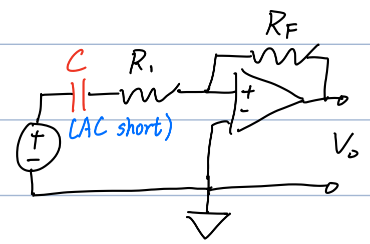
!1. Opamp gain ↓ at high w! (large internal C for stability)

2. DC feedback path!

E. AC coupled amp, C filters DC

$\frac{\underline{V_o}}{\underline{V_I}} = -\frac{\underline{Z_F}}{\underline{Z_I}} = -\frac{R_F}{R_1 + \frac{1}{jwC}} \approx -\frac{R_F}{R_1}$ for $\frac{1}{wC} \gg R_1$

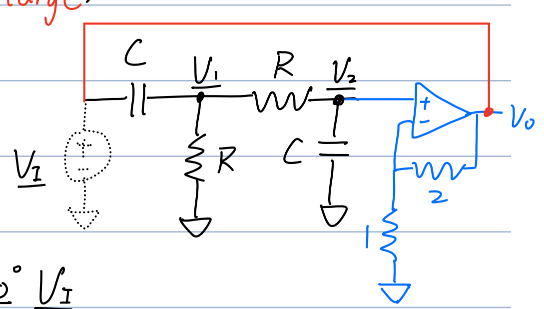
$C \gg \frac{1}{wR_1}$ (large)



E. Feedback oscillator

$$\left.\begin{aligned} &\frac{\underline{V_1} - \underline{V_I}}{1/jwC} + \frac{\underline{V_1}}{R} + \frac{\underline{V_1} - \underline{V_2}}{R} = 0 \\ &\frac{\underline{V_1} - \underline{V_2}}{R} = \frac{\underline{V_2}}{1/jwC} \end{aligned}\right\}$$

$$\underline{V_2} \overset{\cdots}{=} \frac{jwRC}{(1 - w^2R^2C^2) + j3wRC}\,\underline{V_I}$$



$\underline{V_2}$ real when $w = \frac{1}{RC} \equiv w_0 \xrightarrow{\text{where}} \underline{V_2} = \frac{1}{3}\angle 0^\circ\,\underline{V_I}$

Then use n.i. amp to amplify back 3x. → feedback as $\underline{V_I}$, self-driven

AC power

instantaneous $p(t) = v(t)\, i(t)$ (passive)

avg $\overline{p(t)} \equiv P = \frac{1}{T}\int_0^T p(t)\, dt$

Prop. $\overline{kf_1(t)+f_2(t)} = k\,\overline{f_1(t)} + \overline{f_2(t)}$

$\overline{\cos(\omega t+\phi)} = 0$

$\overline{\cos^2(\omega t+\phi)} = \frac{1}{2}$ $\quad \overline{\sin(\omega t+\phi)\cos(\omega t+\phi)} = 0$ (same ϕ !) $\quad (\sin x \cos x = \frac{1}{2}\sin 2x)$

rms $P = \overline{i^2(t)}R = \left(\sqrt{\overline{i^2(t)}}\right)^2 R \rightarrow I_{rms} \equiv \sqrt{\overline{i^2(t)}} = \sqrt{\frac{1}{T}\int_0^T i^2(t)\, dt}$

$= I_{rms}^2 R$

$V_{rms} = I_{rms} R \xrightarrow{\text{ana}} DC$ $\quad P \equiv P_{avg}$ is NOT P_{rms} !

E. sinusoid $i(t) = I_m \cos(\omega t+\phi)$

$I_{rms} = \frac{I_m}{\sqrt{2}}$ $\quad V_{rms} = \frac{V_m}{\sqrt{2}}$

R v/i in phase $p(t) = V_m I_m \cos^2(\omega t) = \frac{1}{2} V_m I_m (1+\cos(2\omega t))$

$P = \frac{1}{2} V_m I_m = V_{rms} I_{rms}$

L i lags v $p(t) = V_m \cos(\omega t)\, I_m \underbrace{\cos(\omega t - 90^\circ)}_{\sin(\omega t)} = \frac{1}{2} V_m I_m \sin(2\omega t)$

$P = 0$ (avg.)

C i leads v $p(t) = \text{— —— —— -} +90^\circ) = -\frac{1}{2}$ ————

$P = 0$ (avg.)

General $p(t) = v(t)\, i(t) = V_m I_m \cos(\omega t + \theta_v)\cos(\omega t + \theta_i)$ $\quad \cos x \cos y = \frac{1}{2}[\cos(x-y) + \cos(x+y)]$

$= \frac{1}{2} V_m I_m [\cos(\theta_v - \theta_i) + \underbrace{\cos(2\omega t + \theta_v + \theta_i)}_{\text{avg} = 0}]$

$P = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i)$

$i(t) = I_m \cos(\omega t + \theta_i) \stackrel{\rightarrow}{=} I_m \angle\theta_i$

$= I_{m1} \angle\theta_v + I_{m2} \angle\theta_v - 90^\circ$

$\stackrel{\text{proj.}}{=} \underbrace{I_m \cos(\theta_v - \theta_i)}_{\text{in-phase}} \cos(\omega t + \theta_v) + \underbrace{I_m \sin(\theta_v - \theta_i)}_{\text{quadrature}} \sin(\omega t + \theta_v)$

$p(t) = v(t)\, i(t) = V_m \cos(\omega t + \theta_v)[I_{m1} \cos(\omega t + \theta_v) + I_{m2} \sin(\omega t + \theta_v)]$

$= V_m I_{m1} \cos^2(\omega t + \theta_v) + V_m I_{m2} \cos(\omega t + \theta_v) \sin(\omega t + \theta_v)$

$= \underbrace{\frac{1}{2} V_m I_m \cos(\theta_v - \theta_i)}_{P} [1 + \cos(2(\omega t + \theta_v))] + \underbrace{\frac{1}{2} V_m I_m \sin(\theta_v - \theta_i)}_{Q} \sin(2(\omega t + \theta_v))$

$P \rightarrow$ average power $\quad Q \rightarrow$ reactive power (borrow/return)

[VAR]

Apparent power $P = V_{rms} I_{rms} \cos(\theta_v - \theta_i)$ $S \equiv V_{rms} I_{rms}$ [VA]

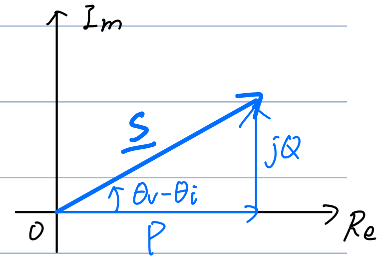
power factor $P = S \cdot pf$, $pf \equiv \cos(\theta_v - \theta_i)$

↳ How efficiently avg. power is absorbed

pf. angle $\equiv \theta_v - \theta_i$ (lag/lead), lost in cos

Complex power $\underline{S} = S \underline{/\theta_v - \theta_i}$, $|\underline{S}| = S = V_{rms} I_{rms}$

$= P + jQ$



$P = V_{rms} I_{rms} \cos(\theta_v - \theta_i)$ $Q = V_{rms} I_{rms} \sin(\theta_v - \theta_i)$

$\underline{S} = V_{rms} I_{rms} \underline{/\theta_v - \theta_i} = V_{rms} \underline{/\theta_v}\ I_{rms} \underline{/-\theta_i} = \underline{V_{rms}}\ \underline{I_{rms}}^*$

where $\underline{V_{rms}} \equiv V_{rms} \underline{/\theta_v}$, $\underline{I_{rms}} \equiv I_{rms} \underline{/\theta_i}$

Passive loads $\underline{Z} = \frac{\underline{V}}{\underline{I}} = \frac{V_m}{I_m} \underline{/\theta_v - \theta_i} = \frac{V_{rms}}{I_{rms}} \underline{/\theta_v - \theta_i} = \frac{\underline{V_{rms}}}{\underline{I_{rms}}}$

$\underline{Z} = R + jX$, same ∡ w/ $\underline{S}$

$\underline{S} = I_{rms}^2 \underline{Z} = I_{rms}^2 R + j I_{rms}^2 X$ $I_{rms} = \frac{V_{rms}}{|\underline{Z}|}$

$P = I_{rms}^2 R$, $Q = I_{rms}^2 X$

	R, P	X, Q	$\theta = \theta_v - \theta_i$	pf	$(P \geq 0)$
pure R	> 0	0	0	1	
L	≥ 0	> 0	$(0, 90°]$	lag	
pure L	0	> 0	$90°$	0	
C	≥ 0	< 0	$[-90°, 0)$	lead	
pure C	0	< 0	$-90°$	0	
active	can < 0				

$\underline{S}$ conservation $\underline{S} = \underline{V_{rms}}\ \underline{I_{rms}}^* = \underline{V_{rms1}}\ \underline{I_{rms1}}^* + \cdots + \underline{V_{rmsn}}\ \underline{I_{rmsn}}^* = \underline{S_1} + \cdots + \underline{S_n}$

$\underline{S} = \sum \underline{S_i}$ cons. of complex power

$P = \sum P_i$ avg.

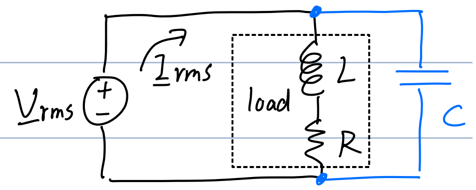
$Q = \sum Q_i$ reactive

pf correction

$P = V_{rms} I_{rms} \cdot pf$ $\qquad pf = \cos(\theta_v - \theta_i)$

If V_{rms} fixed, $I_{rms} \uparrow$ to compensate pf

E. motor w/ high $L \rightarrow$ add C to correct pf.



$$Q = Q_{load} + Q_C$$
$$= Q_{load} - V_{rms}^2 \omega C$$

Don't want to over compensate (pf = 0.95 good)

Max avg. power transfer

given fixed $\underline{Z_s} = R_s + jX_s$

To max $\underline{I_{rms}}$, $I_{rms} = \dfrac{V_{s,rms}}{|Z|} = \dfrac{V_{s,rms}}{\sqrt{(R_s^2+R_L)^2 + \cancel{(X_s^2+X_L^2)}}}$ wont if $X_L = -X_s$

$\rightarrow$ normal match $\rightarrow P_L = I_{rms}^2 R_L$ $\qquad R_L = R_s$

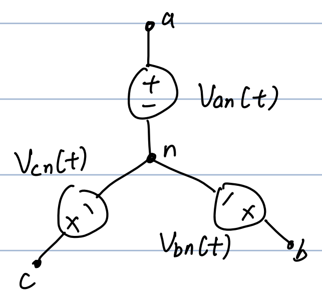
$$\underline{Z_L} = \underline{Z_s}^*$$

* Power is not linear at same ω

if $\omega_1 \neq \omega_2$, $P_1 = \dfrac{V_{rms1}^2}{R}$, $P_2 = \dfrac{V_{rms}^2}{R}$, $P = \dfrac{V_{rms1}^2}{R} + \dfrac{V_{rms2}^2}{R} + \cancel{\dfrac{2 v_1(t) v_2(t)}{R}}$ if $\omega_1 \neq \omega_2$

↳ Then $P = P_1 + P_2$

3-phase (Y)



$V_{an}(t) = \sqrt{2} V_p \cos(\omega t) \quad \leftrightarrow V_p \angle 0°$

$V_{bn}(t) = \sim \cos(\omega t - 120°) \leftrightarrow V_p \angle -120°$ delay 120°

$V_{cn}(t) = \sim \cos(\omega t - 240°) \leftrightarrow V_p \angle -240°$ delay 240°

Now assume $V_p \equiv V_{rms} = \dfrac{\text{peak}}{\sqrt{2}}$.

$\underline{V_{an}} + \underline{V_{bn}} + \underline{V_{cn}} = 0 \rightarrow \Sigma$ in t also 0.

balanced set . same mag, ∡ differ by 120°

Line-to-line voltage

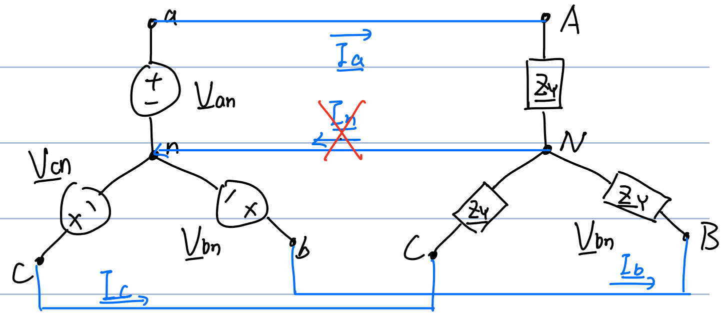
$\underline{V_{ab}} = \underline{V_{an}} - \underline{V_{bn}} = \sqrt{3}\, V_p \angle 30° \equiv V_L \angle 30°$ (law of cosine)

V_L: rms of line voltage $\qquad \underline{V_{bc}} = \qquad V_L \angle -90°$

$= \sqrt{3} V_p \qquad \underline{V_{ca}} \qquad V_L \angle +150°$

$\underline{V_{ab}} + \underline{V_{bc}} + \underline{V_{ca}} = 0 \rightarrow$ also balanced

Balanced Y−Y → same Z_Y (balanced load)



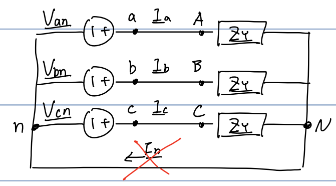
$I_a = \frac{V_{an}}{Z_Y}$ $\quad I_b = \frac{V_{bn}}{Z_Y}$ $\quad I_c = \frac{V_{cn}}{Z_Y}$

All I rotated by same $(-\angle Z_Y)$ → balanced

$I_L = |I_a| = |I_b| = |I_c| = \frac{V_p}{|Z_Y|}$

$I_n = I_a + I_b + I_c = 0$ if balanced (not needed, or much thinner wire)

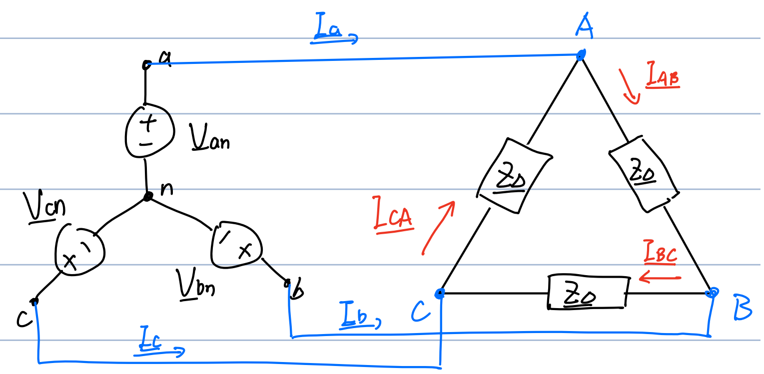
$p(t) = V_{an}(t)\, i_a(t) + V_{bn}(t)\, i_b(t) + V_{cn}(t)\, i_c(t)$



$\overset{\dots}{=} 3 V_p I_L \cos\theta \qquad \theta = \angle Z_Y$

$= \sqrt{3}\, V_L I_L \cos\theta \qquad$ (const!)

Balanced Y − Δ

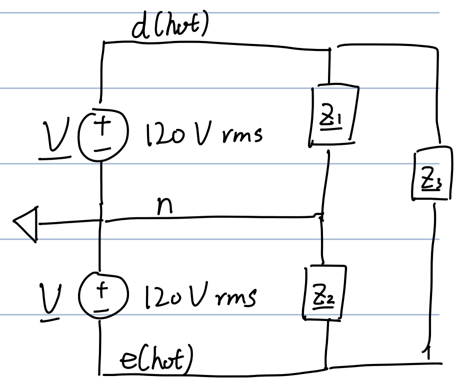


$I_\Delta = |I_{AB}| = |I_{BC}| = |I_{CA}| = \frac{V_L}{|Z_\Delta|}$

$I_a = I_{AB} - I_{CA}$

$I_b = \cdots$, $\quad I_c = \cdots$

E. home Z_3 uses 240 V rms



Ground on metal parts

B coupling – share same flux

$$\phi_1 = \alpha N_1 i_1 + k_m \alpha N_2 i_2 \quad , \quad 0 \le k_m \le 1$$

$$\lambda_1 = N_1 \phi_1 = (\alpha N_1^2) i_1 + (k_m \alpha N_1 N_2) i_2$$

$$\equiv L_1 i_1 + M_{12} i_2$$

$$L_1 \equiv \alpha N_1^2 \qquad M_{12} \equiv k_m \alpha N_1 N_2$$

$$M_{12} \overset{\cdots}{=} M_{21} = M \overset{\cdots}{=} k_m \sqrt{L_1 L_2}$$

$$W = \frac{1}{2} L_1 i_1^2 + \frac{1}{2} L_2 i_2^2 + M i_1 i_2$$

$$v_1(t) = \frac{d\lambda_1}{dt} = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$$

$$\underline{V_1} = j\omega L_1 \underline{I_1} + j\omega M I_2$$

Ideal trans $\quad L_1, L_2 \to \infty \quad , \quad k = 1$

$$\left.\begin{aligned} \frac{V_2}{V_1} &= \frac{N_2}{N_1} = n. \\ \frac{i_2}{i_1} &= \frac{N_1}{N_2} = \frac{1}{n} \end{aligned}\right\} \quad v_2(t)\, i_2(t) = v_1(t)\, i_1(t)$$

No P consumption or storage for ideal

$$\underline{Z_{in}} = \frac{Z_L}{n^2}$$

$\rightarrow$ transform $\underline{V}, \underline{I}, \underline{Z}$!

w-response

$x(t) \rightarrow \boxed{} \rightarrow y(t)$

$X_m \cos(\omega t+\theta_x) \rightarrow \boxed{} \rightarrow Y_m \cos(\omega t+\theta_y)$

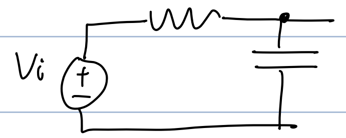
$M(\omega) = \frac{Y_m}{X_m}$ $\quad\quad$ $\phi(\omega) = \theta_y - \theta_x$

$H(j\omega) = \frac{Y}{X}$ $\quad\quad$ $= |H(j\omega)|$ $\quad\quad$ $= \measuredangle H(j\omega)$

V_o/V_i	I_o/I_i	V_o/I_i	I_o/V_i
v-gain	i-gain	transimpedance	transadmittance

E. $H(j\omega) = \frac{V_o}{V_i} \stackrel{\cdots}{=} \frac{1}{1+j\omega RC}$

$M(\omega) = \frac{1}{\sqrt{1+(\omega RC)^2}}$, $\phi(\omega) = -\tan^{-1}(\omega RC)$

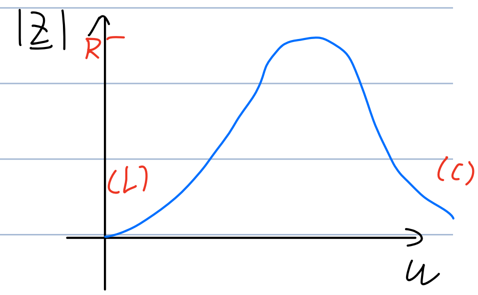


Resonant (parallel RLC)

$\frac{1}{Z} = \frac{1}{R} + j\omega C + \frac{1}{j\omega L}$

$Z = \frac{R}{1+jR(\omega C - \frac{1}{\omega L})}$

$|Z| = \frac{R}{\sqrt{1+R^2(\omega C-\frac{1}{\omega L})^2}}$, $\measuredangle Z = -\tan^{-1}(R(\omega C - \frac{1}{\omega L}))$



when $\omega = \frac{1}{\sqrt{LC}} \equiv \omega_0$, $Z = R$

Drive w/ I_i $\rightarrow$ $V_o = Z I_s$ $\rightarrow$ max output at res!

Quality factor $Q = 2\pi \frac{\text{total w stored}}{\text{w stored w/i 1 period}}$

$= 2\pi \frac{\frac{1}{2} C V_m^2}{\frac{1}{2}\frac{V_m^2}{R}\frac{2\pi}{\omega_0}}$

$Q_0 = R\omega_0 C = R\sqrt{\frac{C}{L}} = \frac{R}{\omega_0 L}$ $\quad\quad$ $Q \uparrow$ as $R \uparrow$

B ω when $|Z| = \frac{|Z_0|}{\sqrt{2}}$ (half-power)

here phase shift is $\pm 45^\circ$ $\xrightarrow{\cdots}$ $\omega_{1,2} = \omega_0\sqrt{1+\frac{1}{4Q_0^2}} \mp \frac{\omega_0}{2Q_0}$

$\omega_1\omega_2 = \omega_0^2$

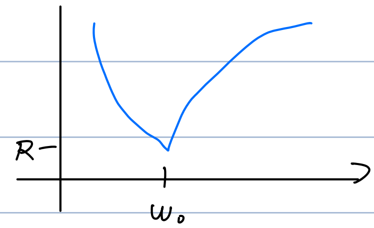
$B = \omega_2 - \omega_1 = \frac{1}{RC} = \frac{\omega_0}{Q_0}$

At resonance. $|I_c| = |I_L| = Q_0|I_i|$

L dominance : $+90^\circ$ (Z)

C $\quad\quad$ -90° (Z)

Series RLC $\quad Z = R + j(\omega L - \frac{1}{\omega C}) \qquad I_o = \frac{V_i}{Z}$



$\omega_o = \frac{1}{\sqrt{LC}}$

$B \doteq \frac{R}{L} = \frac{\omega_o}{Q_o}$

At res, $|V_L| = |V_c| = Q_o |V_i|$

Lossless RLC $\quad \parallel \quad$ reject at $\omega = \omega_o$

$\quad - \quad$ pass at $\omega = \omega_o$

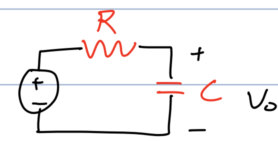
Filters LP, HP, BP, B reject, care about $|H|$

Passive V-div., $H(j\omega) = \frac{V_o}{V_i} = \frac{Z_2}{Z_1 + Z_2}$

Passband at $\omega_p \rightarrow$ make $Z_2(j\omega_p) \rightarrow \infty$ or $Z_1(j\omega_p) \rightarrow 0$

Stopband at ω_s $\quad Z_2(j\omega_s) \rightarrow 0, \quad Z_1(j\omega_s) \rightarrow \infty$

E. 1st order passive LPF $\quad H(j\omega) = \frac{1/j\omega C}{R + 1/j\omega C} = \frac{1}{1 + j\omega RC}$



cutoff $\omega_c \equiv \frac{1}{RC}$ $\quad = \frac{1}{1 + j\frac{\omega}{\omega_c}}$

$M(\omega) = \frac{1}{\sqrt{1 + (\frac{\omega}{\omega_c})^2}}$ $\quad \frac{1}{\sqrt{2}}$ at ω_c, $\quad \phi(\omega) = -\tan^{-1}\frac{\omega}{\omega_c} \longrightarrow -90°$

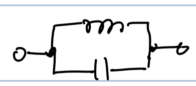
LR $\quad H(j\omega) = \frac{R}{R + j\omega L} = \frac{1}{1 + j\omega \frac{L}{R}}$

$\omega_c = \frac{R}{L}$

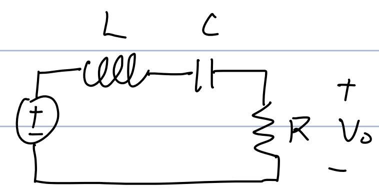
E. ~~ HPF $\quad H(j\omega) = \frac{R}{R + 1/j\omega C} = \frac{j\omega RC}{1 + j\omega RC} \equiv \frac{j\frac{\omega}{\omega_c}}{1 + j\frac{\omega}{\omega_c}}$

$M(\omega) = \frac{\frac{\omega}{\omega_c}}{\sqrt{1 + (\frac{\omega}{\omega_c})^2}} \qquad \phi(\omega) = 90° - \tan^{-1}\frac{\omega}{\omega_c}$

BPF

	$\omega = 0$	$\omega = \omega_o$	$\omega = \infty$
	0	∞	0
	∞	0	∞

E. PBPF $\quad H(j\omega) = \frac{R}{R + j\omega L + 1/j\omega C} = \frac{(j\omega)RC}{1 + (j\omega)RC + (j\omega)^2 LC}$



cutoff $\equiv \omega_1, \omega_2 \qquad \omega_o = \frac{1}{\sqrt{LC}}$

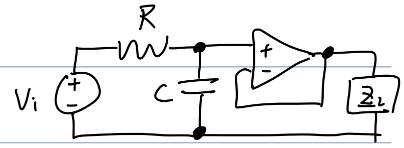
BSF $\quad H(j\omega) = \sim\sim\sim = \frac{1 + (j\omega)^2 LC}{1 + (j\omega)RC + (j\omega)^2 LC}$ (notch filter)

E. Audio crossover

E. bypassing, AC coupling to use C to take hi/DC component so the desired go thru load

E. DC bias + AC signal (DC divider $\xrightarrow{Th}$ R $\Rightarrow$ V_{AC} — C — R analysis)

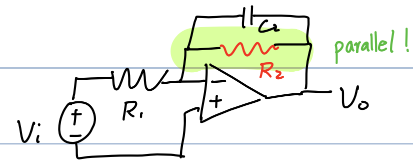
Active 1. buffering $\underline{Z}_L$ interferes passive components.



E. transconductor to conv. $V \to I \Rightarrow$ drive $\parallel$ RLC

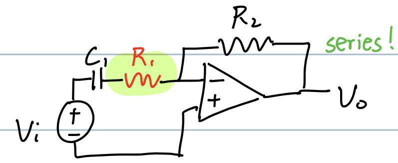
2. Active filter (in. amp) $\frac{\underline{V}_o}{\underline{V}_i} = -\frac{\underline{Z}_2}{\underline{Z}_1}$

E. ALPF $\underline{H}(j\omega) = -\frac{1}{R_1} \cdot \frac{R_2 \cdot 1/j\omega C_2}{R_2 + 1/j\omega C_2} = -\frac{R_2}{R_1} \frac{1}{1+j\omega R_2 C_2} \equiv \frac{k}{1+j\,\omega/\omega_c}$



$k \equiv -\frac{R_2}{R_1}$, can >1, $\omega_c \equiv \frac{1}{R_2 C_2}$

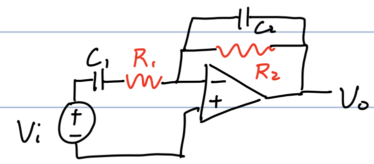
AHPF $\underline{H}(j\omega) = k \frac{j\,\omega/\omega_c}{1+j\,\omega/\omega_c}$



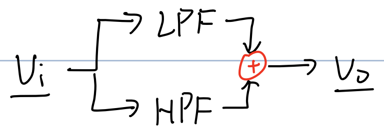
ABPF can do ALPF — AHPF :(

$\underline{H}(j\omega) \overset{\dots}{=} k \frac{j\,\omega/\omega_{c1}}{(1+j\,\omega/\omega_{c1})(1+j\,\omega/\omega_{c2})}$,

broad, need $\omega_{c2} \gg \omega_{c1}$

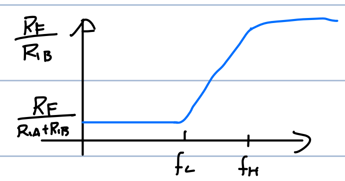


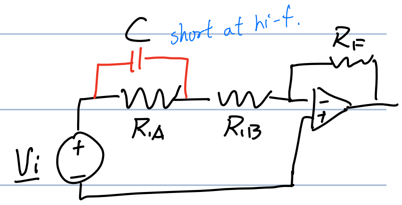
ABSF



summer

Pre-emphasis: pass lo/hi f w/ diff. gain





Scaling 1. impedance (RLC) $\underline{Z}' \to k_z \underline{Z}$ $\quad L' = k_z L$, $C' = \frac{1}{k_z} C$

$\underline{V}, \underline{I}$ ratios don't change, ω_0, B unchanged

2. frequency $\quad \underline{A}(j\omega) \to \underline{A}'(j k_f \omega)$ $\quad L' = \frac{1}{k_f} L \quad C' = \frac{1}{k_f} C$

$\omega_0' \to k_f \omega_0$, amp unchanged

Bode plot
- k
- $j\frac{\omega}{\omega_a}$
- $1 + j\frac{\omega}{\omega_a}$
- $1 + 2\zeta\left(j\frac{\omega}{\omega_a}\right) + \left(j\frac{\omega}{\omega_a}\right)^2$ $\quad b = \frac{2\zeta}{\omega_a} \quad c = \frac{1}{\omega_a^2}$

Log $\quad dB = 10 \log\left(\frac{P_2}{P_1}\right) = 10 \log\left(\frac{V_2^2/R}{V_1^2/R}\right)$

$= 20 \log\left(\frac{V_2}{V_1}\right)$ $\qquad \sqrt{2} \approx 3\,dB$ (V) $\qquad$ (cutoff)

$M_{dB}(\omega) = 20 \log(M(\omega))$

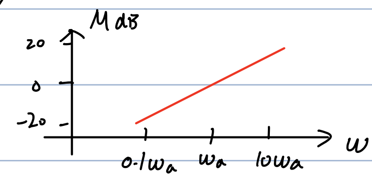
$H^N \rightarrow M^N \qquad \underbrace{N \cdot M_{dB} \qquad N \cdot \phi}_{\text{logged}}$

$H_1 \times H_2 \rightarrow M_1 \cdot M_2 \qquad M_{1\,dB} + M_{2\,dB} \qquad \phi_1 + \phi_2$

1. const. K $\qquad M_{dB} = 20 \log|K| \qquad \phi = 0^\circ \qquad (180^\circ$ if $K < 0)$

2. $j\frac{\omega}{\omega_a}$ $\qquad M_{dB}(\omega_a) = 0 \qquad \phi = 90^\circ\ (j)$

$+20$ dB / decade



3. $1 + j\frac{\omega}{\omega_a}$ $\quad \approx 1, \quad \omega \ll \omega_a$ $\quad$ straight line approx. of 1 and 2.

$\approx j\frac{\omega}{\omega_a}, \quad \omega \gg \omega_a$ $\quad$ 0 dB $\xrightarrow{\text{3dB at } \omega_a}$ lin +20 dB/dec ; $0^\circ \xrightarrow{0.1\omega_a}$ lin $+45^\circ$/dec $\rightarrow 90^\circ$ ($45^\circ, \omega = \omega_a$ $\quad 10\,\omega_a$)

$M_{dB}(\omega) = 20 \log\sqrt{1 + \left(\frac{\omega}{\omega_a}\right)^2} \qquad \phi(\omega) = \tan^{-1}\left(\frac{\omega}{\omega_a}\right)$

4. $1 + 2\zeta\left(j\frac{\omega}{\omega_a}\right) + \left(j\frac{\omega}{\omega_a}\right)^2$ $\quad \approx 1, \quad \omega \ll \omega_a \qquad 0\,dB \qquad 0^\circ$

$\approx \left(j\frac{\omega}{\omega_a}\right)^2, \quad \omega \gg \omega_a \qquad 40\,dB/dec, \quad 180^\circ$ (90°/dec)

bad at $\omega \approx \omega_a$, especially for low ζ, since $H(j\omega_a) \approx 0$ $\qquad \phi$ sharper at low ζ

M_{dB}, ϕ add / $\cdot N$

Opamp gain $\quad A \downarrow$ at hi-ω. Also phase shift $\rightarrow$ $V_o = A(j\omega)V_D$

$= \frac{A_0}{1 + j\frac{\omega}{\omega_b}} V_D$

$\omega = A_0\,\omega_b$: unity gain freq.

Cmplx. ω $\quad v(t) = V_m e^{\sigma t} \cos(\omega t + \phi)$ $\quad$ exp factor

$= \mathrm{Re}\{\underbrace{V_m e^{j\phi}} e^{(\sigma + j\omega)t}\}$ $\quad s \equiv \sigma + j\omega$

$= \mathrm{Re}\{\underline{V} e^{(\sigma + j\omega)t}\}$

$= \mathrm{Re}\{\underline{V} e^{st}\}$

$\omega < 0$, same (cos even)

$x(t) = e^{\sigma t} X_m \cos(\omega t + \phi_X) \rightarrow \boxed{} \rightarrow y(t) = e^{\sigma t} Y_m \cos(\omega t + \phi_Y)$

$\underline{X} = X_m \angle \phi_X \rightarrow \boxed{H(s)} \rightarrow \underline{Y} = Y_m \angle \phi_Y$

E. L, $i(t) = \mathrm{Re}(\underline{I} e^{st})$

$v(t) = L\frac{di}{dt} = \mathrm{Re}(sL\underline{I} e^{st}) = \mathrm{Re}(\underline{V} e^{st})$ $\quad (\underline{V} = sL\underline{I}$ ana. $j\omega L\underline{I})$

C $\quad$ $\underline{V} = \frac{1}{sC} \underline{I}$

Def. Generalized imp $\underline{Z} = \frac{\underline{V}}{\underline{I}}$ $\quad (R, sL, \frac{1}{sC})$, behavior under $|s|$

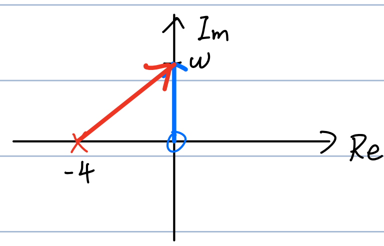
$\underline{H}(j\omega) = \underline{H}(s)|_{s=j\omega}$

3D plot to show $|H(s)|$, σ, ω $\quad$ (prev. we've only looked $\sigma = 0$, $\omega \geq 0$)

General $\underline{H} = \frac{\underline{Y}}{\underline{X}} = \frac{P(s)}{Q(s)} = K_H \frac{(s-z_1)\cdots(s-z_m)}{(s-p_1)\cdots(s-p_n)}$ $\begin{matrix}\rightarrow \text{zero} \\ \rightarrow \text{pole}\end{matrix}$

E. $\underline{H}(s) = \frac{s}{s+4} = \frac{s-0}{s-(-4)}$ $\quad z = 0$, $p = -4$

$\underline{H}(j\omega) = \frac{j\omega - 0}{j\omega - (-4)}$ $\quad$ (length ratio, $\measuredangle$ diff)



$|\underline{H}(j\omega)| = |K_H| \frac{r_1 \cdots r_m}{l_1 \cdots l_n}$

$\measuredangle \underline{H}(j\omega) = \measuredangle K_H + r_1 + \cdots + r_m - l_1 - \cdots - l_n$

Cmplx excitation $\quad \underline{x}(t) \rightarrow \boxed{} \rightarrow \underline{y}(t)$ $\quad \underline{X} e^{st} \longrightarrow \underline{Y} e^{st}$

$\mathrm{Re}[\underline{x}(t)] \rightarrow \quad \rightarrow \mathrm{Re}[\underline{y}(t)]$

DFQ $\quad \underline{x}^{(k)}(t) = s^k \underline{X} e^{st}$ $\quad a_n y^{(n)} + \cdots + a_0 y = b_m x^{(m)} + \cdots + b_0 x$

$\updownarrow$ $\quad (a_n s^n + \cdots + a_0)\underline{Y}\cancel{e^{st}} = (b_m s^m + \cdots + b_0)\underline{X}\cancel{e^{st}}$

$\underline{H}(s)$ $\quad \underline{H}(s) = \frac{\underline{Y}}{\underline{X}} = \frac{b_m s^m + \cdots + b_0}{a_n s^n + \cdots + a_0}$

Natural $a_n y_N^{(n)} + \cdots + a_0 y_N = 0$ Let $y_N(t) = Ae^{\lambda t}$

$(a_n \lambda^n + \cdots + a_0)\cancel{Ae^{\lambda t}} = 0$ roots: λ_ℓ

$y_N(t) = A_1 e^{\lambda_1 t} + \cdots + A_n e^{\lambda_n t}$ (if cmplx., must be conj.)

$H(s)$ poles are natural freqs. except in cancellation

on circuit: deactivate source. Diff. circuits may have same natural ω.

Stability (natural dies out?)

only p matter. z don't.

LT convert circuit to s-domain + init. cond.

3-terminal network E. 

4-port 

admittance/impedence/hybrid params.