$LC$ memory: stores $E \to$ gives out later

**C** $\quad q = Cv$ (linear!), $\quad C = \frac{\varepsilon A}{d} \quad \varepsilon = K\varepsilon_0$

$i = C\frac{dv}{dt} \qquad v = v(t_0) + \frac{1}{C}\int_{t_0}^{t} i(t')dt'$ $\qquad$ passive sign conv.

$= \frac{1}{C}\int_{-\infty}^{t} i(t')dt'$

$v\updownarrow \to i \neq 0 \; ; \quad i \text{ fin.} \to v \text{ cont.}$

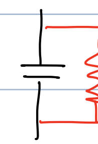
**Energy** $\quad \Delta w = v\Delta q$

$w = \int_0^q \frac{q'}{C}dq' = \frac{1}{2}\frac{q^2}{C} = \frac{1}{2}Cv^2$

**Power** $\quad p = vi = vC\frac{dv}{dt}$

**Charged C** $\quad v(t) = v_0 + \frac{1}{C}\int_0^t i(t')dt'$

**Leakage** $\quad \parallel$ w/ R



**L** $\quad$ Solenoid's $\underline{B}$ leaks on sides, toroid better

$\underset{[\text{Wb}]}{\phi} = \alpha N i \qquad v = N\frac{d\phi}{dt}$

$= \alpha N^2 \frac{di}{dt} \qquad \alpha = \frac{\mu A}{\ell}$ for toroid, where $\ell$ is the mean core perimeter

$\equiv L\frac{di}{dt}$ $\qquad$ and for long solenoid

$\int_{i(t_0)}^{i(t)} di = \frac{1}{L}\int_{t_0}^{t} v(t')dt'$

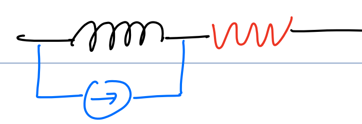
$i(t) = \frac{1}{L}\int_{-\infty}^{t} v(t')dt'$

**E** $\quad p = iL\frac{di}{dt} \qquad w = \int_{t_0}^{t} p(t')dt'$

$= \int_0^i L i' di'$

$= \frac{1}{2}Li^2$

**Real L** $\quad$ series w/ R



**Eq.** $\quad \parallel$ w/ $\textcircled{\uparrow}$

**Combination** $\quad$ same as R, not energy though $\quad$ E. initial charge

$\int \quad V_{out} = -\frac{1}{RC}\int_0^t V_{in}(t')dt'$
$\frac{d}{dt} \quad = -RC\,\dot{V}_{in}(t)$ $\qquad \Big\}$ find $i(t)$

$1^{st}$ order circuits    single E storage element (L/C)

☆ C's $v$ and L's $i$ ⇒ continuous, ⇒ energy

RC  $i_C(t) + i_R(t) = 0$

$$C\frac{dv_C}{dt} + \frac{v_C}{R} = 0$$

$$\tau\frac{dv_C}{dt} + v_C = 0$$

$$v_C(t) = V_0 e^{-t/\tau}$$

$$RC \equiv \tau \equiv \frac{L}{R}$$

RL  $v_L(t) + v_R(t) = 0$

$$L\frac{di_L}{dt} + Ri_L = 0$$

$$\tau\frac{di_L}{dt} + i_L = 0$$

$$i_L(t) = I_0 e^{-t/\tau}$$

General C/L, compute $R_{eq}$. relative to C/L

Driven RC  $v_R(t) + v_C(t) = V_s$

$$\tau\frac{dv_C}{dt} + v_C = V_s \quad (\text{w/ } v_C(0) = V_0)$$

$$v_C(t) = (V_0 - V_s)e^{-t/\tau} + V_s$$

natural : let $V_s = 0$

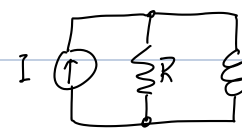
forced: $v_C(t) = V_s$ is a sln.

$$= \underbrace{V_0 e^{-t/\tau}}_{\propto V_0,\ \text{init}} + \underbrace{V_s(1 - e^{-t/\tau})}_{\propto V_s,\ \text{excitation}}$$

$\Sigma$ of causes

RL  $\tau\frac{di_L}{dt} + i_L = I_s$   (parallel)

$$i_L(t) = I_s + (I_0 - I_s)e^{-t/\tau}$$



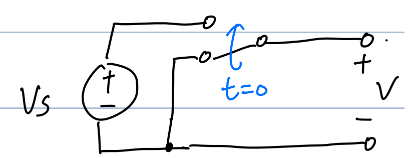

Linear if $V_0 \overset{\text{z.i.}}{=} 0$, else not lin!

General: 1 cap. (L), any R, indep. source → Thevanin (Norton), w/ init. $V_0$ ($I_0$) on cap. (L)

~var: superposition of indep. source & $v_C(t)$, $y(t) = Y_F + (y(0) - Y_F)e^{-t/\tau}$

Steady-state  $y(t) \to Y_F$, —||— → open , —ooo— → short

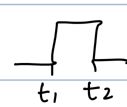
(w/ ◇, $\tau$ can $< 0$, unstable)

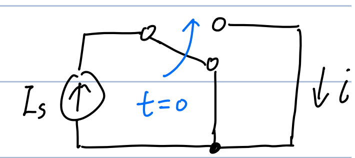


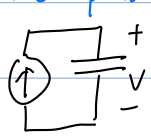

Switch  $v_C(0^-) = v_C(0^+)$. Other quantities may not cont.

Step function  $u(t - t_0)$ ☆

Rec. pulse  $u(t - t_1) - u(t - t_2)$

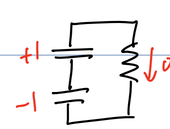


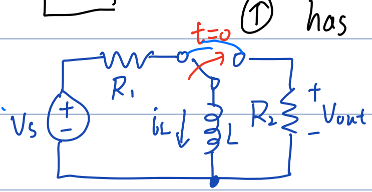

V linearly blows up, marginally stable

Ⓘ has no Th. ~ ∞R, so $\tau \to \infty$, V↑

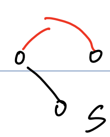



parasitic

E.




H-V pulse generator, $V_{out} = -\frac{R_2}{R_1}V_s$




make-before-break switch to maintain $i$.

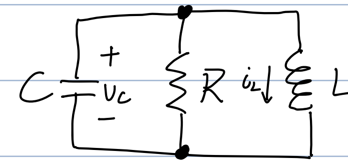
# $2^{nd}$ order circuit

LC  $v_C$, $i_L$ oscillates forever

**Parallel RLC**

$R\uparrow$, loss $\downarrow$

$$\begin{cases} ① \; v_C = L\frac{di_L}{dt} & \text{KVL} \\ ② \; C\frac{dv_C}{dt} + \frac{v_C}{R} + i_L = 0 & \text{KCL} \end{cases} \qquad \begin{cases} ③ \; v_C(0) = V_0 \\ ④ \; i_L(0) = I_0 \end{cases}$$




② → ①: $LC\frac{d^2v_C}{dt^2} + \frac{L}{R}\frac{dv_C}{dt} + v_C = 0$  ④ → ② $C\dot{v}_C(0) + \frac{V_0}{R} + I_0 = 0$

$$\ddot{v}_C + \frac{1}{RC}\dot{v}_C + \frac{1}{LC}v_C = 0 \qquad \alpha \equiv \frac{1}{2RC} \qquad \dot{v}_C(0) = -\frac{1}{C}\left(I_0 + \frac{V_0}{R}\right)$$

$$\ddot{v}_C + 2\alpha\dot{v}_C + \omega_0^2 v_C = 0 \qquad \omega_0 \equiv \frac{1}{\sqrt{LC}}$$

General: $\ddot{y} + 2\alpha\dot{y} + \omega_0^2 y = 0$, init: $y(0) = Y_0$, $\dot{y}(0) = D_0$  Try $y = Ae^{\lambda t}$

$$(\lambda^2 + 2\alpha\lambda + \omega_0^2)\cancel{Ae^{\lambda t}} = 0$$

$$\lambda_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2}$$
$$\lambda_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2}$$

$$y(t) = A_1e^{\lambda_1 t} + A_2e^{\lambda_2 t}$$

$\alpha^2 - \omega_0^2 > 0$  $R < \frac{1}{2}\sqrt{\frac{L}{C}}$  $y = A_1e^{\lambda_1 t} + A_2e^{\lambda_2 t}$  $y(0) = A_1 + A_2$, $\dot{y}(0) = \lambda_1A_1 + \lambda_2A_2$  overdamped

$\alpha^2 - \omega_0^2 = 0$  $R = \frac{1}{2}\sqrt{\frac{L}{C}}$  $y = (A_1t + A_2)e^{-\alpha t}$  $y(0) = A_2$, $\dot{y}(0) = A_1 - \alpha A_2$  critically damped

$\alpha^2 - \omega_0^2 < 0$  $R > \frac{1}{2}\sqrt{\frac{L}{C}}$  $y = e^{-\alpha t}(B_1\cos\omega_d t + B_2\sin\omega_d t)$  underdamped

$y(0) = B_1$  $(\dot{y}(0) = \omega_d B_2 - \alpha B_1)$  $\omega_d \equiv \sqrt{\omega_0^2 - \alpha^2}$

$i_L$ ① → ②  $\ddot{i}_L + \frac{1}{RC}\dot{i}_L + \frac{1}{LC}i_L = 0$  same coeff as $v_C$ !!

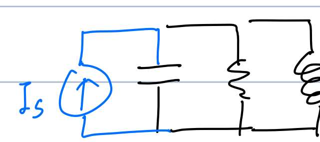
$\dot{i}_L(0) = \frac{V_0}{L}$  init. differ!

**Series RLC**

$$\begin{cases} C\frac{dv_C}{dt} = i_L \\ L\frac{di_L}{dt} + Ri_L + v_C = 0 \end{cases} \qquad \ddot{i}_L + \frac{R}{L}\dot{i}_L + \frac{1}{LC}i_L = 0 \qquad \begin{matrix} i_L(0) = 0 \\ \dot{i}_L(0) = -\frac{1}{L}(V_0 + RI_0) \end{matrix}$$

$\alpha = \frac{R}{2L}$  $\omega_0 = \frac{1}{\sqrt{LC}}$

**Driven parallel**




$$L\dot{i}_L = v_C$$
$$C\dot{v}_C + \frac{1}{R}v_C + i_L = I_s$$
$$\ddot{i}_L + \frac{1}{RC}\dot{i}_L + \frac{1}{LC}i_L = \frac{1}{LC}I_s \qquad \text{same init. cond.}$$

Forced: open C, short L, $\longrightarrow i_{L,F} = I_s$

so $y(t) = y_F + y_n(t) \rightarrow$ 3 cases, and find coeffs.

Series $C\dot{v}_c = i_L$

$L\dot{i}_L + Ri_L + v_c = V_s \rightarrow \ddot{v}_c + \frac{R}{L}\dot{v}_c + \frac{1}{LC}v_c = \frac{1}{LC}V_s$

Summary 2 indep. elements

Diff circuits may look same when it comes to natural response

Switched $v_c(0^+) = v_c(0^-)$, $i_L(0^+) = i_L(0^-)$

Solve for init cond at $0^-$ (steady)

Lossless LC ($R \to \infty$ for parallel)

$\alpha = 0$, $\omega_d = \omega_0 = \frac{1}{\sqrt{LC}}$

App. oscillator, make a $-G$ via transconductor