

Passive sign convention: current enter at $\oplus$

$$\Delta q = i \Delta t$$

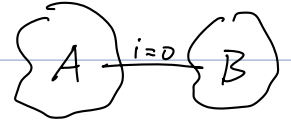
$$p(t) = \frac{dw}{dt} = \frac{dw}{dq} \frac{dq}{dt} = v(t) i(t)$$

Assumptions 1. Lumped action is instantaneous

2. No ext. fields (E/M)

3. Electrical neutrality

KCL Two parts connected via only 1 wire, $i=0$



~~E~~ antenna, not lumped (too long)

Node a region of circuit w/ connections

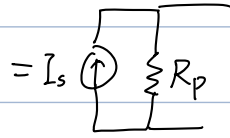
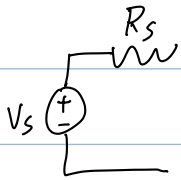
Circuit w/ n nodes $\rightarrow n-1$ indep. KCL

1. $\sum \text{enter}$ 2. $\sum \text{leave}$ 3. $\sum = \sum$

KVL 1. $\sum = 0$ 2. $\sum_1 = \sum_2$

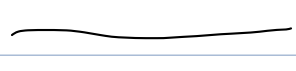
#KCL + #KVL = #elements

Ext eq. cir.



with $R_s = R_p$
 $i_s = \frac{V_s}{R_s}$

⊗ - 2 v.s. in parallel (eq. short) (if $V_s = \infty$ s/ns.)

- 2 i.s. in series ($\sim$ open) ()

Wheatstone bridge: 2 v dividers

Active circuits - Linear dependent

VCVS voltage controlled v. source

$$\mu V_c$$

v. gain

VCCS

$$g_m V_c$$

trans conductance

CCCS

$$\beta i_c$$

i. gain

CCVS

$$r_m i_c$$

trans resistance

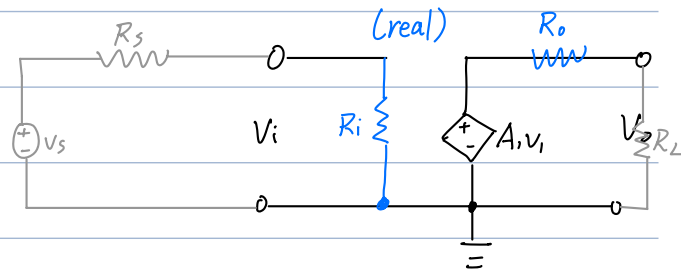
if indep. = 0 $\rightarrow$ dep. = 0

Voltage amp., linear region & saturation

saturation $\rightarrow$ distortion

$$V_I = \frac{R_i}{R_s + R_i} V_s \rightarrow \text{Need } R_i \gg R_s$$

$$V_O = \frac{R_L}{R_o + R_L} A_V V_I \rightarrow \sim R_o \ll R_L$$



OpAmp V_o come from $\pm V_{cc}$

$$V_o = A V_D = A (V_p - V_n) \quad \text{At linear}$$

ideal: input $i \rightarrow 0$

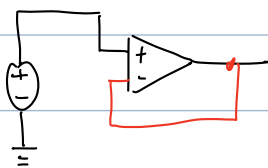
$$R_o \rightarrow 0$$

Negative feedback ensures $V_n = V_p$

$$A \rightarrow \infty$$

V_o, i_o no limit

buffer



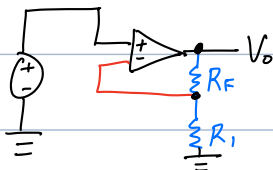
$$V_D = V_i - V_o = V_i - A V_D$$

neg. feedback

$$V_D \approx 0$$

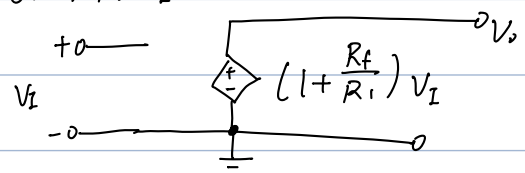
$$V_o \approx V_i \quad \left(= \frac{A}{A+1} V_i \right)$$

Ni. amp.

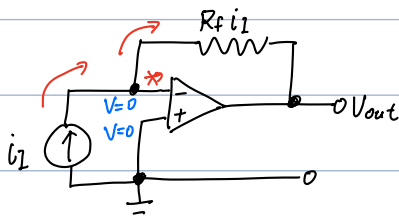


Voltage divider

$$V_o = \left(1 + \frac{R_f}{R_i} \right) V_i \quad \text{ratio, makes it (E. temp.) indep.}$$



$i \rightarrow v$ converter

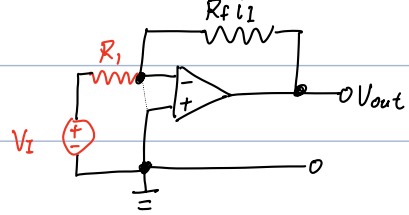


$$V_o = -R_f i_i$$

E. transducer, photodiode

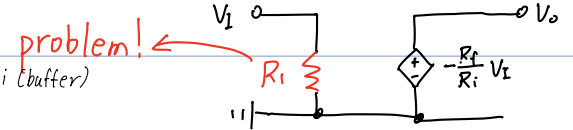


I. amp.

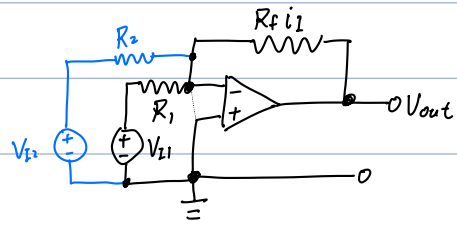


$i_I \rightarrow V_I, R_I$ $V_O = -\frac{R_f}{R_I} V_I$

Typically start with no (buffer)



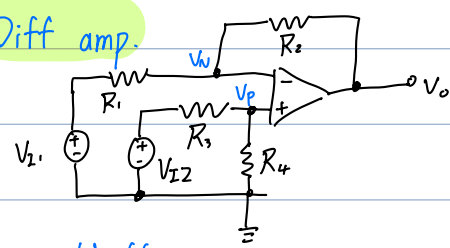
Sum amp.



Combine currents $V_O = -(\frac{R_f}{R_I1} V_{I1} + \frac{R_f}{R_I2} V_{I2})$

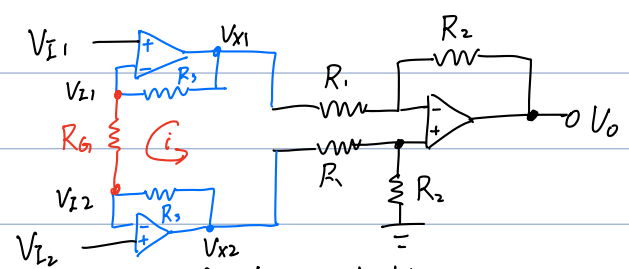
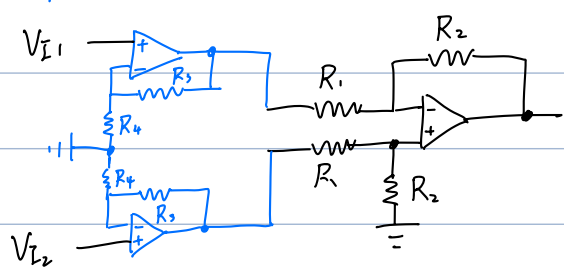
Casade gain X

Diff amp.



KCL $\frac{V_{I1} - V_N}{R_I1} = \frac{V_N - V_O}{R_f}$
 $V_P = V_{I2} \frac{R_4}{R_3 + R_4}$ If $\frac{R_3}{R_4} = \frac{R_I1}{R_I2}$, $V_O = \frac{R_f}{R_I1} (V_{I2} - V_{I1})$
 $V_N = V_P$

w/ buffer:

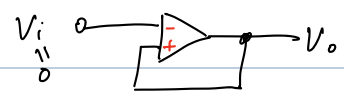


instrumentation amp.

$i = \frac{V_{N1} - V_{N2}}{R_G} = \frac{V_{I1} - V_{I2}}{R_G}$
 $V_{X1} - V_{X2} = (R_G + 2R_3) i$
 $= (1 + \frac{2R_3}{R_G}) (V_{I1} - V_{I2})$
 $V_O = -\frac{R_f}{R_I1} (1 + \frac{2R_3}{R_G}) (V_{I1} - V_{I2})$

Positive feedback

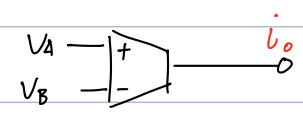
blow up! (until sat.)
 unstable eq.



Comparator no feedback, saturated, faster

Transconductor VCCS, $i_o = G_m (V_A - V_B)$

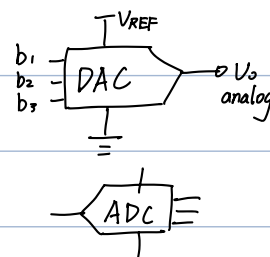
E. transistor, but idealized linear relation



D → A converter (sum amp.), V_o is inverted!

A → D converter compare V_i with diff. refs. (E. 3-bit: $0.5V \sim 6.5V$)

⇒ logic converter → output



Systematic tech.

Node voltage analysis E. circuit w/ $\oplus$, $\sim$. Use KCL on every node

E. book p 228,
$$\begin{bmatrix} \frac{1}{R_1} + \frac{1}{R_2} & -\frac{1}{R_2} \\ -\frac{1}{R_2} & \frac{1}{R_2} + \frac{1}{R_3} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} i_{s1} \\ i_{s1} + i_{s2} \end{bmatrix}$$

std. form
$$\begin{bmatrix} G_1 + G_2 & -G_2 \\ -G_2 & G_2 + G_3 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} i_{s1} \\ i_{s1} + i_{s2} \end{bmatrix}$$

unknown node v indep. i. sources

$$\underline{G} \underline{V} = \underline{i}_s$$

$$\underline{V} = \underline{G}^{-1} \underline{i}_s$$

Cramer's rule $\Delta = G_1 G_2 + G_2 G_3 + G_3 G_1$

To find V_2 , find Δ_2 by replacing 2nd col. of $\underline{G}$ by $\underline{i}_s$ and find det

$$\Delta_2 = \begin{vmatrix} G_1 + G_2 & i_{s1} \\ -G_2 & -i_{s1} + i_{s2} \end{vmatrix} = -G_1 i_{s1} + (G_1 + G_2) i_{s2}$$

$$V_2 = \frac{\Delta_2}{\Delta} = \frac{-G_1}{G_1 G_2 + G_2 G_3 + G_3 G_1} i_{s1} + \frac{G_1 + G_2}{G_1 G_2 + G_2 G_3 + G_3 G_1} i_{s2}$$

+ dep. source $\uparrow$ 1. VCCS (similar)

2. CCCS write i of interest in terms of node voltages

+ voltage source (don't know i thru $\oplus$)

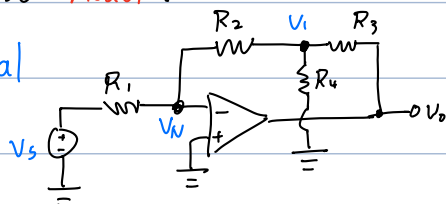
Already gives an eq. relating 2 node V .

Create supernode by finding all i crossing the $\oplus$ and any $\parallel R$.

+ dep. voltage source $\diamond$ (same thing)

+ OpAmp Use model!

E. ideal



KCL

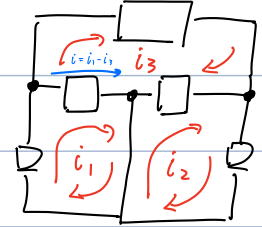
$$G_1(V_N - V_s) + G_2(V_N - V_1) = 0$$

$$G_2(V_1 - V_N) + G_4 V_1 + G_3(V_1 - V_o) = 0$$

$V_N = 0$ due to negative feedback

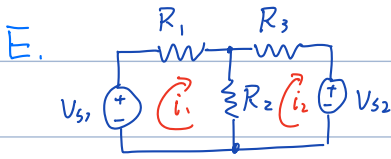
$$V_o = -V_s \left(\frac{R_2}{R_1} + \frac{R_3}{R_1} + \frac{R_2 R_3}{R_1 R_4} \right)$$

Mesh-current analysis - only valid for planar circuits (no ∇)



Mesh: loop w/o any inner loops

Unknown: mesh currents (usually same dir.)



KVL $-V_{s1} + R_1 i_1 + R_2 (i_1 - i_2) = 0$ $\Rightarrow (R_1 + R_2) i_1 - R_2 i_2 = V_{s1}$
 $V_{s2} + R_2 (i_2 - i_1) + R_3 i_2 = 0$ $\Rightarrow -R_2 i_1 + (R_2 + R_3) i_2 = V_{s2}$

$\underline{R} \underline{i} = \underline{V_s}$

* if see a $-$ in $(\oplus)$, put a $-$, when thru R , use $+$
 $+$ $\uparrow$, predefine an i branch, create **supermesh** skipping $\uparrow$

Node ∇

Mesh i

planar only!

good for
solving for

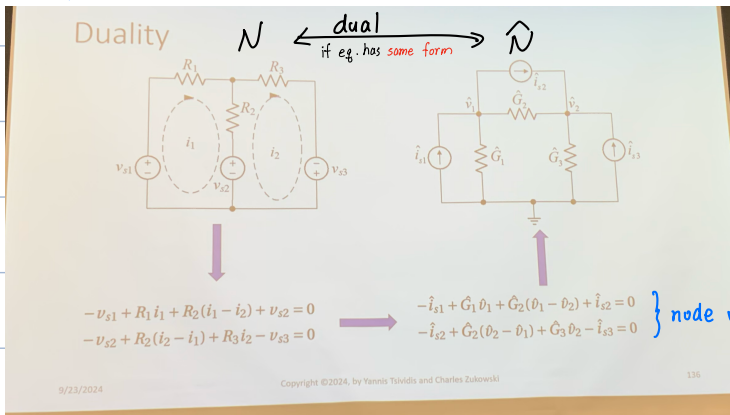
parallel
 ∇

series
 i

Always simplify!

Duality

node $\leftrightarrow$ mesh



$V \rightarrow i$, $R \xrightarrow{\Omega} \hat{G} \xrightarrow{S}$, series $\rightarrow$ parallel

exact dual if same numerical value

Pathological

$\Delta = 0$, or overconstrained

put small R in series w/ $(\oplus)$, large R $\parallel$ w/ $(\uparrow)$

Linear circuit prop. & thm

$$y = \underbrace{C_1 x_{s1}}_{y_1} + \dots + \underbrace{C_k x_{sk}}_{y_k}$$

output → indep. sources (inputs)

Proportionality & superposition

$$y = \sum_{k=1}^K (\text{output w/ only one indep source on})$$

Kill: Short $\oplus$, open $\uparrow$

Eg. R only R and $\diamond$

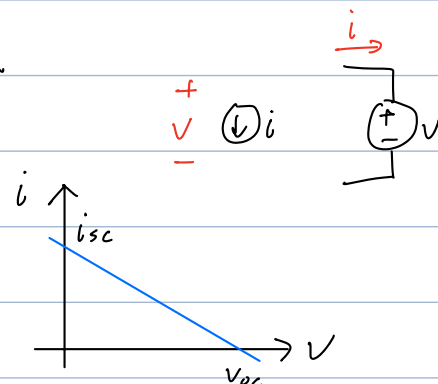
$$V = R_{eq} i$$

+ indep. sources $\bigcirc$ By conv., i is leaving the $+$ term.

V_{oc} voltage when terminal of interest is open

i_{sc} current when terminal is shorted

Two-term i - V char



Thévenin V_{oc} series w/ R_{Th} → R_{eq} , killing all indep. sources w/

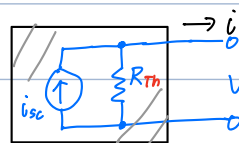
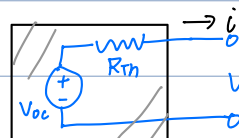
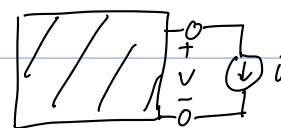
Pf. Superpos Drive circuit by i

1. ext. source = 0, $V = V_{oc}$

2. int. ^{indep.!} source = 0, $V = R_{Th} (-i)$

1+2

$$V = V_{oc} - R_{Th} i$$



Norton i_{sc} parallel w/ R_{Th}

$$R_{Th} = \frac{V_{oc}}{i_{sc}}$$

T vs N